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COLLECTION OF MECHANISMS FOR 10 LINKS KINEMATIC CHAINS OF GROUP G

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Abstract:-*in this paper a new, easy, reliable, and efficient method to detect isomorphism and a catalogue of fixed link with its corresponding equivalent links in the distinct mechanisms of kinematic chains of Group-G, has been presented. It is helpful to the new researchers and designers to choose the best mechanism to perform the desired task at the conceptual stage of design. The proposed method is presented by comparing the structural invariants 'sum of the absolute values of the characteristic polynomial coefficients' [SCPC] and 'maximum absolute value of the characteristic polynomial coefficient' [MCPC] of [JJ] matrices. These invariants are used to detect isomorphism in the mechanism kinematic chain having simple joints. The method is explained with the help of examples of planar kinematic chain.*

Keywords: *Kinematic Chain; Fixed Link; Equivalent Link; SCPC; MCPC*

Notations used: *F.L.-Fixed Link, E.L.-Equivalent Link, $n_5\, n_4\, n_3\, n_2$ –Pentagonal, Quaternary, Ternary, Binary Links*

1. INTRODUCTION

Over the past several years much work has been reported in the literature on the structural synthesis of kinematic chains and mechanisms. Undetected isomorphism results in duplicate solutions and unnecessary effort. Therefore, the need for a reliable and efficient algebraic method for this purpose is necessary. Identifying isomorphism among kinematic chains using characteristic polynomials of adjacency matrices of corresponding kinematic chains are simple methods (Uicher &Raicu 1975), (Mruthyunjaya & Raghavan 1979), (Yan & Hall, 1982), (Raicu,1974),(Tuttle,1975). But the reliability of these methods was in questions as several counter examples were found by ([Mruthyunjaya, 1984),(Mruthyunjaya & Balasubramanium,1987). The test proposed by Mruthyunjaya [Mruthyunjaya, & Balasubramanium 1987] is based on characteristic coefficients of the 'Degree matrix' of the graph of the kinematic chains. The elements of the degree matrix were sum of the degree of vertices (degree or type of links) or unity in a linklink adjacency matrix. Later on, this test was also found unreliable. The representation polynomial is the determinant of the generalized adjacency matrix, called representation matrix of the kinematic chain. But the representation matrix requires the use of a large number of symbols, the calculation and comparison of the representation polynomials is not as easy as that of the characteristic coefficients of the adjacency matrix. The reliable proceduresareproposedfor the purposeby (Mohd,1998),(Rao,1989,1997,2000),(Srinath &Rao,2006),Rao&Rao,1993,(Rao,Pratap&Deshmukh,2001),(Yadav,Pratap &Agrawal,1996),(Yan &Hwang1991),(Zhang &Lio 1999), (Hasan,2007) and by several other researchers . Several other methods like (Hasan A., 2009, 2010, 2012,2013), ()Agrawal & Rao,1987),(Ambeker &Agrawal,1987), (Gibson & Marsh ,1989) are also in use.

II. The Joint-Joint [JJ] Matrix

This matrix is based upon the connectivity of the joints through the links and defined, as a square symmetric matrix of size N x N, where n is the number of joints in a kinematic chain.

$$[JJ] = \left\{ L_{ij} \right\}_{N \times N} \quad \text{--------} \quad (1)$$

Where

$$L_{ij} \left\{ \begin{array}{l} = \text{Degree of link between } i^{th} \text{ and } j^{th} \text{ joints} \\ \text{those are directly connected} \\ =0, \text{ if joint i is not directly connected to joint j} \end{array} \right\}$$

Off course all the diagonal elements $L_{ii} = 0$

III. Characteristic polynomial of [jj] matrix

D (λ) gives the characteristic polynomial of [JJ] matrix. The monic polynomial of degree n is given by equation (2).

$$|(JJ - \lambda I)| = \lambda^{n} + a1\lambda^{n-1} + a2\lambda^{n-2} + \text{---------} \; an - 1\lambda + an \quad \text{---------------} \quad \mathbf{(2)}$$

Where; n = number of simple joints in kinematic chain and 1, a_1, a_2, a_{n-1}, a_n are the characteristic polynomial coefficients. The two important properties of the characteristic polynomials are

(a) The sum of the absolute values of the characteristic polynomial coefficients (SCPC) is an invariant for a [JJ] matrix. i.e.
$|1| + |a_1| + |a_2| + \text{-----} + |a_{n-1}| + |a_n| = \text{invariant}$

(b) The maximum absolute value of the characteristic polynomial coefficient (MCPC) is another invariant for a [JJ] matrix.

IV. Structural invariants [scpc] and [mcpc]

The values of characteristic polynomial coefficients are invariants for a [JJ] matrix. To make these [JJ] matrix characteristic polynomial coefficients as a powerful single number characteristic index, new composite invariants have been proposed. These invariants are **'SCPC'** and **'MCPC'**. These invariants are unique for a [JJ] matrix and may be used as identification numbers to detect the isomorphism among simple jointed kinematic chains. The characteristic polynomial coefficients values are the characteristic invariants for the kinematic chains. Many investigators have reported co-spectral graph (non-isomorphic graph having same Eigen spectrum). But these Eigen spectra (Eigen values or characteristic polynomial) have been determined from (0, 1) adjacency matrices. The proposed [JJ] matrix provides distinct set of characteristic polynomial coefficients of the kinematic chains having co-spectral graphs. Therefore, it is verified that the structural invariants **'SCPC' and 'MCPC'** are capable of characterizing all kinematic chains and mechanisms uniquely. Hence, it is possible to detect isomorphism among all the given kinematic chains.

V. Isomorphism of kinematic chains.

Theorem: Two similar square symmetric matrices have the same characteristic polynomials.

Proof:

Let the two kinematic chains are represented by the two similar matrices A and B such that $B = P^{-1} AP$, taking into account that the matrix λI commutes with the matrix P and $|P^{-1}| = |P|^{-1}$. Since the determinant of the product of two square matrices equals the product of their determinants, we have

$$| B - \lambda I | = | P^{-1} A P - \lambda I |$$

$$= | P^{-1}(A - \lambda I) P |$$

$$= | P^{-1} | | (A - \lambda I) | | P | = | A - \lambda I |$$

Hence, D (λ) of 'A' matrix = D (λ) of 'B' matrix.

D (λ) =characteristic polynomial of the matrix.

It means that if D (λ) of two [JJ] matrices representing two kinematic chains is same, their structural invariants 'SCPC' and 'MCPC' will also be same and the two kinematic chains are isomorphic otherwise non-isomorphic chains.

VI. EXAMPLE- 1

Considering two kinemtic chains with 10 bars, 12 joints, three degree of freedom as shown in Fig 1 .The task is to examine whether these two chains are isomorphic. The structural invariants of these two chains are as follows:
For chain 2(a): **[**SCPC] = 8.3734e+006 , [MCPC] =3.5938e+006
For chain 2(b): **[**SCPC] = 7.0147e+006 , [MCPC] = 2.9393e+006
Our method reports that chain 2(a) and 2(b) are non-isomorphic as the set of values of [SCPC] and [MCPC] are different for both the kinematic chains. Note that by using other method comments [Cubillo and Wan, Jinbao, 2005], the same conclusion is obtained.

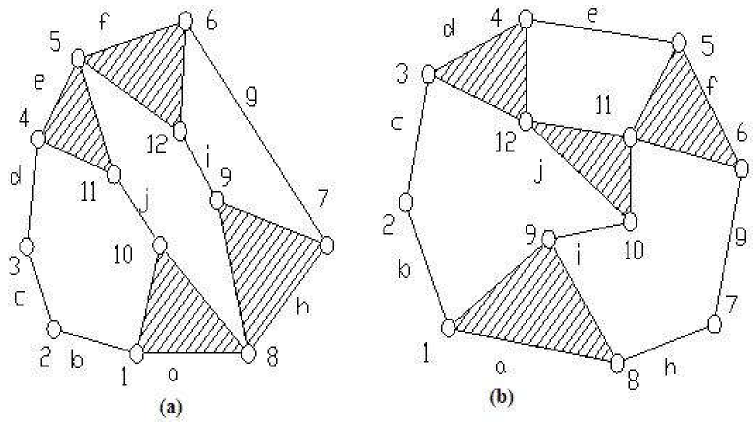
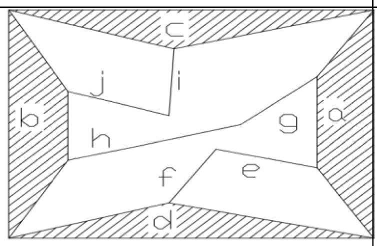
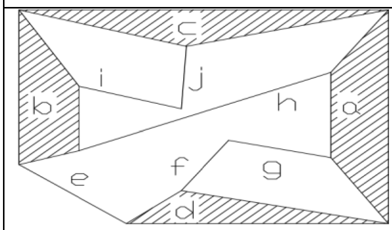
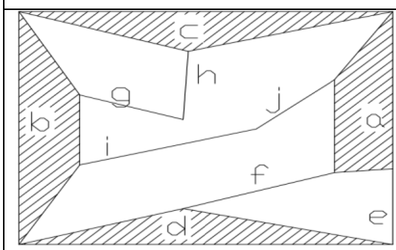
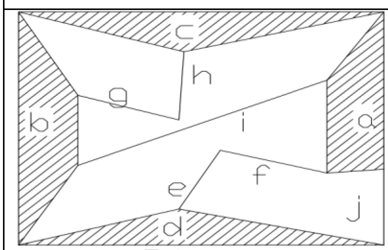


Fig 1: Ten bar KC with 3 dof

VIII. Results

The proposed invariants [SCPC] and [MCPC] are used as the identification number of the kinematic chains having simple joints. The identification numbers of all 1-dof kinematic chains up to 10-Links are with the author .These invariants are also able to detect isomorphism among the kinematic chains with multiple joints also. Fixed Links and Equivalent Links in Distinct Mechanisms of 10 Links, 13 Joints, Single degree of freedom Kinematic Chains GROUP G are listed in Table 1.

Table-1: Mechanisms of 10 links, 13 Joints, 1F Kinematic Chains GROUP G

Kinematic Chain	n5 n4 n3 n2	F. L.	E. I.	D. M.
 G1	**0226**	a c e f g	b d j i h	**5**
 G2	**0226**	a b c d e f g h i j	- - - - - - - - - -	**10**
 G3	**0226**	a b c d e f g h i j	- - - - - - - - - -	**10**
 G4	**0226**	a b c d e f g h i j	- - - - - - - - - -	**10**

X. Conclusions

In this paper, a simple, efficient, and reliable method to identify isomorphism is proposed. By this method, the isomorphism of mechanisms kinematic chains can easily be identified. It incorporates all features of the kinematic chains and as such, violation of the isomorphism test is rather difficult. The method has been found to be successful in distinguishing all known 16 kinematic chain of 8-links, 230 kinematic chain of 10-links having 1-F. The advantage is that they are very easy to compute using MATLAB software. It is not essential to determine both the structural invariants to compare two chains, only in case the [SCPC] is same then it is needed to determine [MCPC]for both kinematic chains. The [JJ] matrices can be written with very little effort, even by mere inspection of the chain. The proposed test is quite general in nature and can be used to detect isomorphism of not only planar kinematic chains of one degree of freedom, but also kinematic chains of multi degree of freedom. All the results of distinct mechanisms obtained from 10 links, 13 Joints, 1F Kinematic Chains GROUP G are listed in Table- 1.

References

[1]. Agrawal V.P. and Rao J.S. (1987). The Mobility Properties of Kinematic Chains", Mech. Mach. Theory, 22, 497-504.

[2]. Agrawal V.P.and Rao J.S. (1987). Structural Classification of Kinematic Chains and Mechanisms, Mech. Mach. Theory, 22, 489-496.

[3]. Ambeker A. G. and Agrawal V. P. (1987). Canonical Numbering of Kinematic Chains and Isomorphism Problem: Min. Code", Mech. Mach theory, 22, No 4, 453-461.

[4]. Ambeker A. G. and Agrawal V. P.(1987).Identification of Kinematic Chains, Mechanisms, Path Generators and Function Generators Using Min. Codes, Mech. Mach theory, 22(4), 463-471.

[5]. Gibson C.G. and Marsh D(1989) .On the Linkage Varieties of Watt 6-Bar Mechanisms-I. Basic Geometry" Mech. Mach. Theory, 24, 105-113.

[6]. Hasan A.(2007). Systematic Development of Kinematic Chains and Mechanisms from a given Assortment of Links,Journal of Institution of Engineers (India), 88,15-19.

[7]. Hasan A. (2007). Isomorphism in Kinematic Chains Using Path Matrix, Journal of Institution of Professional Engineers , New Zealand,1-4.

[8]. Hasan A.(2009). Isomorphism and Inversions of Kinematic Chains up to 10 Links, Journal of 'Institution of Engineers (India), 90, 10-14.

[9]. Hasan A.(2009). Identification of Isomorphism among Kinematic Chains and Inversions Using Link Adjacency Values International J. of Mech. and Materials Engineering 4(3), .309-315.

[10]. Hasan A., (2010). Application of Link Adjacency Values to Detect Isomorphism among Kinematic Chains",Int. J. Mech. Mater. Design, , 6,157-162.

[11]. Hasan A.(2011). Isomorphism Identification of Compound Kinematic Chain and Their Mechanism, imanager's Journal on Mechanical Engineering, 2(1), 7-15.

[12]. Hasan A.(2013). Determination of the Distinct Mechanisms from a Given Kinematic Chain, imanager's Journal on Mechanical Engineering, 3(3), 8-14.

[13]. Min Hwang and Yii – Wen Hwang (1992). Computer Aided Structural Synthesis of Planar Kinematic Chains with Simple Joints, Mech. Mach. Theory, 27, 189 – 199.

[14]. Mohd A. (1998) .Kinematic chains structural modeling and correlation with machine performance, M Tech. Dissertation, IIT Delhi.

[15]. Mruthyunjaya T. S. (1984). A Computerized Methodology for Structural Synthesis of Kinematic Chains, Part – 1, Formulation", Mech. Mach. Theory, 19(6), 487 – 495.

[16]. Mruthyunjaya T. S.(1984). A Computerized Methodology for Structural Synthesis of Kinematic Chains, Part – 2, Application to several fully or Partial Known Cases, Mech. Mach. Theory, 19(6), 497 – 505.

[17]. Mruthyunjaya T.S. and Balasubramanium H.(1987). In Quest of Reliable and Efficient Computational Test for Detection of Isomorphism in Kinematic Chains, Mech. Mach theory, 22(4), 131139.

[18]. Mruthyunjaya T. S. and Ruthven M. R. (1979).Structural Analysis of Kinematic Chains and Mechanisms based on Matrix representation", Transaction of the ASME, Journal of Mechanical Design, 101, 488 – 518.

[19]. Raicu A. (1974). Matrices Associated with Kinematic chains from 3 to 5 Members", Mech. Mach. Theory, 9, 123 – 129.

[20]. Rao A.C. (1989). Selection of Ground, Input and Output Link in Mechanisms, A Quantitative Approach, Trans, CSME, 13(1/2), 23-30.

[21]. Rao A. C (1997). Hamming Number Technique – 2, Generation of n planer kinematic Chains, Mech. Mach. Theory, 32(4), 489-499.

[22]. Rao A. C (2000)]. Application of Fuzzy Logic for the Study of Isomorphism, Inversions, Symmetry, Parallelism and Mobility in Kinematic Chains, Mech. Mach. Theory, 35, 1103-1116.

[23]. Srinath A. and Rao A.C. (2006).Synthesis of Planar Kinematic Chains, Institution of Engineers (I), 86, 195-201.

[24]. Rao A. C. and Rao C. N(1993). Loop Based Pseudo Hamming Value-I, Testing Isomorphism And Rating Kinematic Chain., Mech. Mach. Theory, 28(1), 113- 127.

[25]. Rao A. C. and Rao C. N(1993). Loop Based Pseudo Hamming Value-II, Inversions, Preferred Frames and Actuators, Mech. Mach. Theory, 28(1), 129- 143.

[26]. Rao A. C., Pratap and B. Deshmukh (2001).Computer Aided Structural Synthesis of Planer Kinematic Chains Obviating the Test of Isomorphism. Mech. Mach. Theory, 36, 489-506.

[27]. Tuttle E.R (1975). Generation of Planar Kinematic Chains, Mech. Mach. Theory, 31, 729 – 748.

[28]. Uicker J. J. and Raicu A. (1975). A method for the Identification and Recognition of Equivalence of Kinematic Chains, Mech. Mach. Theory, 10, 375 – 383.

[29]. Yadav J. N. , Pratap C. R.and Agrawal V. P. (1996), Mechanisms of a kinematic chain and degree of structural similarity based on the concept of link – path code", Mech. Mach. Theory, 31, 865 – 871.

[30]. Yan H. S. and Hall A. S. (1982). Linkage Characteristic Polynomials: Assembly Theorems, Uniqueness, ASME Journal of Mechanical Design, 104, pp. 11-20.

[31]. Yan and Hwang (1991). Specialization of Mechanisms, Mech. Mach. Theory, 26 (6), 341-351.

[32]. Zhang W. J. and Li Q.(1999). A New Approach to Mechanism Topology Identification, ASME, Journal of Mechanical Design, 121, pp. 57-64.