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THE HOW, ENERGY FROM CHAOS BECOMES DISCRETE - MONADS

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Abstract:-

From Mechanics in case that , *in an Axisymmetric Rotating Body , with constant Angular-Velocity* , w , ***the moment of Inertia ,*** $J_x = J_y$ ***,*** *is equal about the two of the three Principal axis ,* ***then*** ,

1.. *The Angular – velocity - vector , $\bar{w}$,* describes the Ellipsoid of Angular Velocity and its nib describes a Cone of which Plane-Base is Fixed , ***and Simultaneously*** ,

2.. *The Angular-Momentum , $\bar{B}$,* describes the Ellipsoid of Angular-Momentum and its nib describes a Cone also of which Plane-Base is also Fixed .

3.. *The Nib of Angular – velocity - vector* , $\bar{w}$, describes on the Tangential-Plane of the Angular-Momentum-Ellipsoid , ***the Herpolhode*** , while ,

4.. *The Nib of Angular-Momentum* , $\bar{B}$, describes on the Tangential-Plane of the Angular-Velocity-Ellipsoid , ***the Polhode*** .

5.. *The Fixed-Tangential-Planes on , $\bar{w}$, and , $\bar{B}$, nib* are alternately Perpendicular to , $\bar{B}$, and , $\bar{w}$, central axes of rotation .

All above happen in , Material-Point , where the Positive ⊕ constituent , is Eternally self-rolling on the Negative ⊖ constituent , with Angular-Velocity , w , in Infinite Spherical traces , either at Great -circles or Small-circles , or any other close Spherical-curve , and by Applying all laws of Mechanics into this Energy - Chaos , is thus created the → First – Discrete – Energy - monad , which is the Material Point i.e. , the Quantum of Physics and , of all Energy- Space - Universe .

Present Article [61] *is the completion of prior [60] using the , Vector analysis of Euler-Lagrange.* Everything in this cosmos, is Done or Becomes , from a Mould .

Geometry has the Monad , *the discrete continuity AB* , Becoming from the *Zero-Point* ≡ 0 , and ***Mechanics-Physics*** the Recent-Acquisition of The Material-Geometry , where Zero-point 0 = □ = {⊕+⊖} = The Material-point = *The Quantum* = Positive Space and Negative Anti-Space . [58] ***Monad in Geometry → Linearly*** is , through mould of Parallel Theorem [44-45] ,*which are the equal distances between points of parallel and line →* ***In Plane*** is through mould of Squaring the circle [46-47] , *where the two equal and perpendicular monads consist a Plane acquiring the common Plane- meter* ,π, → ***In Space*** (volume) is through mould of the Duplication of the Cube [44-46] *, where any two Unequal perpendicular monads acquire the common Space-meter ,*$\sqrt[3]{2}$ *, to be twice each other .* ***Monad in Mechanics*** *and Physics* is → The Material-point = *discrete continuity*

|{⊕+⊖}|=

The Quantum through mould of Space –Anti-space in itself , *which is the material dipole in inner monad Structure and is Identical with the Electromagnetic cycloidal field of Energy monads .This is ,*the distance *, the deep concept of Material-geometry.*

Energy monads presuppose Energy-Space Base (*the beyond Planck`s length ,Gravity`s and Spaces` levels*) the [PNS] Space Anti-Space as work → W = ∫ P.ds = 0 , which is the cause of Spaces existence and the motion of particles . Since are also Quantized then , this property is encountered in

Stationary waves where energy , **E** , is proportional to angular velocity **w** . This property of particles , *Angular momentum = Spin* , becomes from the Intrinsic , *Inward* , cycloidal wave motion, which is their cause of external motion *as outward waves* .

The varying lever arms ,on cycloid-evolute is the cause of vibrations and which cause the EM-waves and Spin . Common-circle of radius , r_c , is the common source of vibration excitation for the Space , Anti –space , considered as rotating with angular velocity **,w**, and then their relative motion becomes the , Rolling of Space ABC on Anti-space $A_EB_EC_E$ and since also this relative motion is applied on STPL [Six Triple Points Line] Mechanism , then D_A ,P_A , points on it are the corresponding linear links of vibrations and Poles of rotation .

[STPL] is a ***Geometrical Mechanism*** that produces and composite all opposite Space and Anti-space Points to Material-points → Waves ← the three Breakages { [$s^2 = \pm (\overline{w}.r)^2$, [□i] = $2(wr)^2$] of [MFMF] mechanism under $\overline{v} = \overline{c}$ thrust } , and through it are becoming

***The Fermions →* [$\pm\overline{v}.s^2$] and *The Bosons* → [$\overline{v}$.∇i = [$\overline{v}.2(\overline{w}.r)^2$] = [$\overline{v}.2s^2$]** , [35]

It was shown [33-36] that Un-clashed Fragments through center , O , consist the Medium-Field Material-Fragment →[± s^2] = [MFMF] as base for all motions , and Gravity as force [$\overline{\nabla}$i] , while the clashed with the constant velocity ,$\overline{c}$, consist the Dark matter [± $\overline{c}$.s] and the Dark energy [$\overline{c}.\overline{\nabla}$i] , or from → Breakages [± $s^2 = \pm (wr)^2$] and [$\overline{\nabla}$i = $2(wr)^2$] , where then become Waves { *Distance* ds = AA_E *is the Work embedded in monads and it is what is vibrated* } with Vibrating equations of motion , become ,

A → Particles, with Inherent Vibration ,
B → Gravity-Field-Energy , without Vibration
C → Dark-matter-Energy constituents and as ,
A.. [$\pm\overline{v}.s^2$] → Fermions and → [$\overline{v}$.□i] → Bosons,
B.. [± s^2] → [MFMF] Field, and the binder , Field is [□i] → Gravity force ,
C... [$\pm\overline{c}.s^2$] → Dark matter, and the binder Gravity force [□i] , [$\overline{c}$.□i] → The Expanding Dark energy.

From above is seen that in ,
A *, and ,* C , *case → Energy as velocity ,* $\overline{v}$ *, exists in the Discrete monads ,* ± $\overline{v}.s^2$ *and* ± $\overline{c}.s^2$.
B ***, case , → is the transportation of Energy , from Chaos to Material points*** **[+s² ↔ - s²] .**

5 The How Energy from Chaos Becomes Discrete - Monads NEWTON – FORCES Newton`s , First - Law states that , *Any change in motion involves an acceleration , a* . In circular motion , for an object of mass , m , acceleration is equal to , $a = \frac{v^2}{r}$ and force , F, acted is

$F = m.a = m\frac{v^2}{r}$, which is the Centripetal force F_p . From Newton`s Third-Law , *All forces in the universe occur in equal but opposite directed pairs* , then For any Centripetal force F_p , there is a force of equal magnitude but of opposite direction , the Centrifugal force , F_f , which acts back on the object , *without specifying the nature or origin of the forces* .

In Material-point , [⊕⊖] , both forces exist , ***apriori*** , as , the Glue-Bond between the two opposites which is the main Stress $\sigma = \pm \frac{2.v}{(1+\sqrt{5})}$, and since v = w.r = 2πr/ T = (2π.r) .f , where r = the radius of the Energy cave f = the frequency of this rotation , *then* $\boldsymbol{\sigma = \pm \frac{4\pi r}{(1+\sqrt{5})} . f}$ or → $\mathbf{f = \frac{\sigma\,(1+\sqrt{5}\,])}{4\pi r}}$ **.**

i.e. a relation between the Glue-Bond , **σ** , and the frequency , **f** , of the rotation , or , In Chaos r = r → 0 between the ⊕ , ⊖ , Opposites , exists a Stress , σ , The Centripetal F_p , and Centrifugal force , F_f , which nature is only the frequency in a complete rotation , and

from Planck`s equation **E = h.f** = $\frac{h(1+\sqrt{5}\,]).\sigma}{4\pi r}$ = $\frac{\mathbf{h(1+\sqrt{5}\,])}}{\mathbf{4\pi}}$ **.** $\left[\frac{\sigma}{r}\right]$, ***from Chaos*** , r = r → 0 , becomes the Monad , [⊕⊖] , which is the Neutral – Material – Point . A wide analysis in [58] .

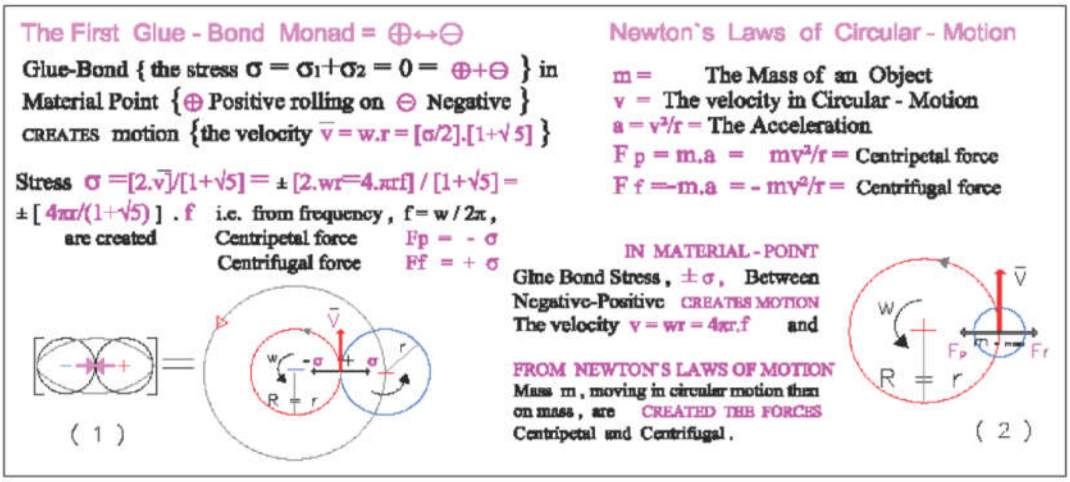


Figure..M0.. In (**1**) The Glue-Bond pair of opposites [⊖ ⊕] , creates rotation with angular velocity w = v/r , and velocity v = w.r = $\frac{2\pi}{T}$ = 2πr.f = [$\frac{\sigma}{2}$].$(1+\sqrt{5}$) or frequency f = $\frac{(1+\sqrt{5}\,]).\sigma}{4\pi r}$, Period T = $\frac{4\pi r}{\sigma(1+\sqrt{5})}$ where ± σ are the two Centripetal F_p and Centrifugal F_f forces . In (**2**) Mass , m , of an object rotating with velocity , v , in a cave of radius , r , creates a pair of equal and opposite forces the Centripetal F_p and Centrifugal F_f .

CONTENT

A... *Preliminaries.*
1. Introduction
Zeno`s Paradox and the nature of Points.

Word , *quantization* , has to do with the discrete continuity, which describes the Physical reality through the Euclidean conceptual , for Points Straight lines, Planes , the Monads in Universe and the Dual Nature of Spaces as *discrete* and *continuous* **.** Euclidean Geometry is proved to be the *Model of Spaces* and Material Geometry the *Model of Physical Reality* since it is Quantized as the Complex numbers , which are such. ***The proposed Euclidean solution.***
Straight line AB is continuous in Points between A and B [i.e. all points between line segment AB ***are the elements which fill AB*** , *and which Points are also Nothing* , or Everything else and are Anywhere as in above and for Achilles in order to run the 100 m, has to pass the infinite points between point A and point B [1.1] . A point ,T, is on line AB only when exists TA + TB = AB (*or the whole* AB *is equal to the parts* TA , TB , *as it is the logic of equality and the logic for equations*) . Since in nature exists the Principle of Equality and Un-equality consequently any Comparison is including the following three cases .
AT < → AB , which become the quantized material- lengths and have ***infinite Spaces , Anti-Spaces and Sub-Spaces*** , then is impossible in--bringing Achilles to the Tortoise´s starting point B , and also for
Tortoise´s to 110 m , because as follows **,**

Straight line AB is not continuous unless* a Common Dimensional Unit AT > 0 *or* AT = ds → AB *is accepted and thus in this way exists ,

a.. Straight line AB is continuous with points as filling (*Infinitively divisible*)
b.. Straight line AB is discontinuous (*discrete*) with dimensional Units , ds, as filling (that is made up of finite indivisible parts the Monads , ds ≠ 0 , as in Material geometry) defining the Space Anti-space at
A,B points and Sub-space as [ds ≠ AB /n , where n =1,2,→∞)

c.. Straight line AB is continuous in ,ds, with ds = 0 as points of filling , and also discontinuous (*discrete*) with the dimensional Units , ds ≠ 0 , defining the Space , Anti-space at A,B points and Sub-space , where , *ds* = *quantum* = AB / n , **{** where n = 1,2,3 → ∞ ,

= [a+b.i] /n = complex number *and Infinitively divisible which is keeping the conservation of Properties at End Points A , B* **}** as filling , ***and cotinuous with points as filling***

1.. In case TA + TB > AB then point T *(for n = ∞ then ds = 0 i.e. the ∞ Positions* is not on line AB , **it is OUT**, and then issues *of points in ds)* , i.e.

the Property of ***Anequality*** and it is the triangle ABT lying in ABT Plane . This is the main difference between the Euclidean and the Non-Euclidean geometries . On this is based the Philosophy of Parallel fifth Postulate which is proofed to be a Theorem and also all the Ancient unsolved and now solved problems . [44-47] ***In Euclidean Geometry points A , B , T consist the Plane ABT , while for Others is a curve in Plane ABT .***
The Definition 2 (*a line AB is breathless length*) is altered as → *for any point* T *on line* AB *exists* TA + TB = AB i.e. it is the equation which is also and equality . [9-10] ***Since points have not any dimension and since only AB has dimension*** (the length AB and for any $\overline{AT}$ the length AT) and since also on $\overline{AB}$ exist infinite line segments Monads ds = 0 → ∞ are simultaneously (*actual infinity*) and also (potential infinity) in Complex number form , and this defines that, infinity exists between all points which are not coinciding , and because ,**ds,** comprises any two edge points with imaginary part then this property differs between all the infinite points .
This is the Vector relation of Monads , ds , (or , as Complex Numbers in their general form $\bar{w} = a + b \cdot i$ ***) , which is the Dual Nature of lines (discrete as*** $\frac{\bar{w}}{|a^2+b^2|}$ ***and continuous as points (.) and in recent Material-Geometry the Work ≡ Energy ≡ Monads ≡ Imaginary part ,i,) .*** [57-58] **2..** In case TA + TB = AB then point T is **ON** straight line AB where then issues the Property of Equality. as the Quanta of Energy , the monad , and is represented as above Dipole i.e.

This motion is Continuous and occurs on Dimensional Units , ds , which is the Maxwell`s Monads-Displacement-Electromagnetic-current [E+∇xP] , and not on Points which are dimensionless , upon these Bounded States of [PNS] , the Spaces and Anti-Spaces , and because of the different Impulses PA , PB , of edge points A , B , and that of Impulses , PiA , PiB of Sub-Spaces , they are either on straight lines AB or on tracks of the Spaces , Anti-Spaces and Sub-Spaces of AB . The range of Relative velocities is bounded according to the single slices of spaces (ds) . [14 -15] , [39-40] . concept of Material-Geometry is this , distance , the First-Self-rotating–Monad in M-Geometry .

1.1. **Achilles and the Tortoise :**

The Problem :

(0m) → (100 m) (110 m)
A -------------------T----------------- B → -----

< In a race , the Quickest runner , Achilles , can neve overtake the Slowest , Tortoise , since the Pursuer must first reach the Point whence the Pursued started , so that the Slower must always hold a lead >
This problem was devised by Zeno of Elea to support Parmenides´s doctrine that < all is one
On Monad AB which maybe equal to → 0 ↔ AB ↔ ± ∞ ← exists < a bounded State of energy for each of the Infinite Spaces and Anti-Spaces called Energy monad in Space moulds > and this [Dipole AB = Matter = The meter of the reaction to Energy-change] is the communicator of Impulse [Force P] of The Definition 2 , (a line AB is breathless length) is altered as , for any point T on line AB exists Equality TA + TB = AB .

The critic of all above is in my articles , and because of the inattention in the establishment in these Definitions , allowed the creation of Non-euclid Geometries which acted Negatively*in Euclidean Absolute Space* > , contrary to the

3.. In case TA + TB < AB then point ,T, is evidence of our senses for plurality and change **IN** straight line AB , *where then is NOT issuing* and to others arguing the opposite . Zeno´s *the Property of Equality or Un-equality* . arguments are as proof by contradiction or It is issuing a ***New Paradox in Geometry*** (*reduction ad absurdum*) which is a which is the recently new Material-Geometry philosophical dialectic method . Achilles at as in articles [55-56] and connects , point ,A, allows the Tortoise at point ,T, a Geometry-Mechanics–Chemistry-Physics. head start 100 m and each racer starts running at some constant speed , *one very fast* From D. Hilbert`s → 4. Problem of the straight *and one very slow* , the Tortoise say has further line as the shortest distance between two points 10 m at point ,B, . A and B become the following :
Since Straight line AB is continuous with points

Lobachevsky : (Hyperbolic Geometry) is as filling , *The Quickest* , has to pass *Infinitive* excluding the axiom of parallels or assume it as points to reach point T , so since the steps are not satisfied . thepoints ($\frac{AB}{\infty}$ = 0) ,*The Quickest* will never
Rieman`s : (Elliptic Geometry) is reach point T . The same also for *The Slower* excluding the axiom of parallels , assuming that one and only one Point lies between the other with step , ($\frac{TB}{\infty}$ = 0) will never reach point B . two.
Hilbert`s : (Non-Archimedian Geometry) is excluding the axiom of parallels , assuming that Infinitive Points on Parallels lie between the other two and straight line is the shortest distance between the two points .
Euclid`s-Markos : (Geometry - Material Geometry) ,

1.2. The Arrow Paradox (Arrow) :
The Problem :

< If everything when it occupies an equal Space is at rest , [PNS] , and if that which is in locomotion is always occupying such a Space at any moment , the flying Arrow is Therefore motionless > | (10 m) | ***Numerical order A → B which is not***
(10m) **ds = a+b.i = v.dt** (10 m) ***valid for Temporal order (dt) . In case that***

A ---------------------> B
A ------- C - D -------> B

The Arrow Paradox is not only a simple mathematical problem , because is referred also to ***motion in Absolute Euclidean Space ,*** i.e. in a Space where ***issues Geometry*** , *with all the unsolved till recently problems as ,The Parallel Postulate the Squaring of circle etc.*, and also the ***Physical*** where Space [PNS] is not moving and because of its Duality (*discrete and continuous*
as Complex numbers are) , shows that , ***Time is not existing as any essence but only a measure for measurements , a number*** .
This Paradox ***is not*** in metaphysical sphere of mind since is was proved in [15] that , Complex numbers and Quantum Mechanics Spring out of ***the Quantized Euclidean Geometry .***
As before Straight line AB is discontinuous (***discrete***) with dimensional Units , **ds=CD** as filling and ***continuous with points*** as filling (The Complex Numbers in the general form w = a+b. i) , ***which is the Dual Nature of lines*** (line = *discrete with , Line-Segments , and continuous with points*).
It has been shown that PNS Primary Neutral Space is not moving and Time is not existing , *so Points* , in Primary Space cannot move to where they are because are already there and motion is impossible . Since any *Points C , D* of the Primary Neutral Space , PNS , are motionless (v = 0) this is at any Time (the composed instants are dt = 0) , and so then motion is impossible , i.e. issues [ds = a + b. i = v.dt] where , *for* $a = 0$ then ds = b.i = v.dt *and for* b ≠0 and dt = 0 then *ds = Constant = = v . 0 →* i.e. **v = ∞ ,** *For b = 0* then ds = a = v.dt *and for* dt = 0 then → *ds = a = Constant = v . 0 →* i.e. again , **v = ∞ ,**

Therefore in PNS , v = ∞ , T = 0 , meaning infinite velocity and Time not existing , so
Since Arrow is moving from point A to point B , then exists the dt = 0 then motion from Point A to point B has not any concept , and the distance CD and anywhere exist the Equal CD is unmovable **, i.e.**
Motion of points C , D of PNS* is not existing *because time (dt = 0) and infinite velocity (v = ∞) exists , while motion of the same points C , D* exists *in PNS out of a moving Sub-Space of AB **(** arrow CD is one of the ∞ roots of AB) **where ,** **(** *ds = CD = The Monad in PNS* **).** [15] .
It has been shown that Primary Neutral Space [PNS] is not moving and Time is not existing , *so Points* , in Primary Space cannot move , *to where they are* , because are already there and motion is impossible. Since *Points T , C,,,* of Primary Neutral Space ,
PNS , are motionless (v = 0) at any Time (the composed instants are dt = 0) then motion (*s=v.dt*) is impossible . i.e. *In PNS v = ∞ and Time = 0 , meaning*
infinite velocity ,v, and Time is not existing , so since any Arrow (a vector) moving from point A to point B , then exists a Numerical order A → B which is not valid for Temporal order (dt) . In case dt = 0 then motion from Point A to point B has not any concept , and distance , *CD magnitude ,* and anywhere exist the Equal CD it is unmovable (*s=v*) ,
i.e. *The Motion of points C , D , T..... of PNS* is not existing *because time (d t = 0) and for ds = Any constant exists with infinite velocity (v = ∞) while motion of the same points*
C , D ,T exists *in PNS out of a moving Sub- Space of AB* (Included Arrow CD is one of the ∞ roots of line segment AB). Monads ds =CD= 0 → ∞ are Simultaneously, *actual infinity* (*because for n = ∞ then ds* = [*AB / (n = ∞)*] = 0 *i.e. a point*) *and , potential infinity* , (*because for n = 0 then ds* = [*AB / (n = 0)*] = ∞ *i.e. the straight line through sector AB* .
*Infinity exists between all points which are not coinciding , and because Monads ,*ds, *comprises any two edge points with Imaginary part , then this property differs between the ,i, infinite points or as* $\overline{ds}$ = λi + □i , which
(λ≡ Space, i≡ Energy) , is the Quaternion .

Since Primary point ,A, is the only Space then on this exists the Principle of Virtual Displacements W = $\int_A^B P.\,ds$ = 0 or [ds.(P_A+P_B)
= 0] , i.e. for any monad ds > 0 Impulse P = (P_A+P_B) = 0 and [ds. (P_A+P_B) = 0] , Therefore , Each Unit AB = ds > 0, exists by this Inner Impulse (P) where $P_A+P_B= 0$, → i.e. The Position and Dimension of all Points which are connected across the Universe and that of Spaces exists , because of this equilibrium Static Inner Impulse , on the contrary should be one point only (Primary Point A ≡ Black Hole → ds=0 and P = ∞).[17,22] . Monad AB is dipole [{A(P_A) ←0→(P_B)B}] and it is the symbolism of the two opposite forces (P_A) , (P_B) which are created at points A,B . This Symbolism of primary point (*zero 0 is nothing*) shows the creation of Opposites , A and B , points from this zero point which is the Non-existence. [13] .
All points may exist with force P = 0 → { PNS the Primary Neutral Space} and also with P ≠ 0 , ($P_A+P_B= 0$) , { PS is the Primary Space} for all points in Spaces and Anti – Spaces , therefore [PNS] is self-created , and because at each point

may exist also with $P \neq 0$, then [PNS] is a (perfectly Homogenous , Isotropic and Elastic Medium) Field with infinite points (i) which have a ± Charge with force $Pi = 0 \rightarrow P = \Lambda \rightarrow \infty$ and containing everything .
Since points A ,B of [PNS] coincide with the infinite Points , of the infinite Spaces , Anti-Spaces and Sub-Spaces of [PNS] and exists there rotational energy $\pm \Lambda$ and since Motion may occur at all Bounded Sub–Spaces ($\pm \Lambda , \lambda$), then this Relative motion is happening between all points belonging to [PNS] and to those points belonging to the other Sub-Spaces (A≡B) . The Infinite points in [PNS] form infinite Units (The monads = segments) AiBi = ds , which equilibrium by the Primary Anti-Space by an Inner Impulse (P) at edges A , B where $P_{iA}+P_{iB} \neq 0$, and $ds = 0 \rightarrow N \rightarrow \infty$.
Monad , *discrete* , (Unit ds = Quaternion) $\overleftrightarrow{AB}$ is the ENTITY and [AB – P_A , P_B] is the LAW , therefore Entities are embodied with the
Laws . Entity is quaternion $\overleftrightarrow{AB}$, and law |AB| = Energy length (*the energy quanta*) of points |A,B| or the wavelength where then AB=0 and imaginary part are the equal forces
P_A , P_B as the fields , *the medium* , in monads ,

(*This is distinctly seen for* ***Actions at a distance*** *, where there the continuity of all intermediate points being also nothing ,* ***is succeeded on a*** *quantized , tiny energy volume which consists* **the material point** *i.e.*
A field , the medium , ***or*** *by the Exchange of energy in the Inner-monads field*) . [39-40]. Pythagoras definition for a **Unit →** *it is a Point without position while* **a Point → is the Material Point** *which is a Unit having position .*

1.3. The dichotomy Paradox (***Dichotomy*** **) :**

The Problem :
< ***That which is in locomotion must arrive at the half-way stage before it arrives at the goal*** **>**

(0m) → → (50 m) (100 m)
A ---------D------------ C------------------- B

As before Straight line AB is not continuous unless a Common Dimensional Unit AC > 0 or ds = 0 → AB / 2 → AB is accepted and this because point C is on line AB where then issues CA + CB = AB and since CA = CB then CD < CB therefore point D on (AD) will pass through C on(AC) before it arrives at the goal B on (AB) .

1.4. The Algebraic Numbers :

From priors *Monad* ≡ **AB = 0 ↔ AB ↔ ± ∞ , is and also represents** the Spaces ,A, the Anti-Spaces ,B, Sub-Spaces of AB which are **the Infinite Regular Polygons** , on circle with AB as Side , and on circle with AB as diameter , and it is what is said , *monad in monad* . According to De Moivre's formula the n-th roots on the unit circle AB are represented by the vertices of these Regular n-sided Polygon inscribed in the circle which are Complex numbers in the general form as , $\mathbf{w = a+b.i = r.e^{(i\varphi)}}$, and , **a** and **b** = Real Numbers , $r = \sqrt{a^2+b^2}$, (±) **i** = Imaginary Unit. We will show that since Complex Numbers are ***on Monad AB*** (*A Monad is any two points non coinciding are monads*) and it is the only manifold , for the Physical reality , ***so then Euclidean Geometry is also Quantized*** .
This geometrically is as follows,

a. Since Exists $\sqrt[2]{1} = \pm 1$ or square roots of monad are **[-1 ↔ +1]** , therefore **xx** (axis) coordinate system represents the one-dimensional Space (+1) and the
Anti-Space (-1) which is (the Straight line) , 1.1 = 1 , (-1).(-1) = 1 **[+ i]**

b. Since Exists $\sqrt[2]{-1} = \pm i$ **or [↨]** , therefore **yy** (axis) coordinate system represents [- i] a perpendicular on (-i).(- i) = + i ² = + (-1) = -1 , (+ i).(+ i) = + i ²
= -1 meaning that *Euclidean -Geometry is such Quantized , which is the Energy-Space* . [15]
i.e. The Position and Dimension of all Points which are connected across the Universe and that of Spaces exists , because of this Static Inner Impulse P , **on the contrary** ***should be one point only (Primary Point ≡ Black Hole → ds = 0) .*** [43 - 45]
Impulse is ∞ and may be Vacuum ,

c. Since Exists $\sqrt[3]{1}$ **=** the three roots **[1 ,**
- ½ + (√ 3.i) /2 , - ½ - (√ 3.i) /2] therefore **xx-yy** coordinate system represents the two - dimensional ± Spaces and ± Complex numbers , (the Plane)
1.1.1 = 1 , [-½ +(√3.i) /2 **]** ³ = 1 , [-½
-(√3.i) /2] ³ = 1 + i

d. Since Exists $\sqrt[4]{1} = \sqrt[2]{\sqrt[2]{1}} = \sqrt[2]{\pm 1} =$ **[+1 , -1] , [** $\sqrt[2]{-1}$ **= + i , - i]** or -1 ↔ +1 , ↨ therefore coordinate systems **xx - yy** represent all these Spaces , - i , (± Real and ± Complex numbers) , where Monad = 1 = (*that which is one*) , represents the three-dimensional Space and
Anti-Space (the Sphere) which is , $[\pm 1]^4 =$ $= [\pm i]^4 = 1$.

The fourth root of **1** ***are the vertices of Square in circle*** with **1** as diameter and this because the Geometrical Visualization of Complex numbers , is the formula **$\sqrt[4]{1} = \pm 1 , \pm i$** …(d) and also since ± 1 is the one-dimensional ***real Space*** (*the straight line*) , the vertical axis is the other one-dimensional ***Imaginary Space*** ± i **.**
Since for dimension , *discrete* , are needed N+1 points , then (d) is representing the Space with three dimensions (dx ,dy ,dz) which are Natural , Real and Complex . Monads (The Entities = AB) are the Harmonic repetition of their roots , and since roots are the combinations of purely real and purely Imaginary numbers , *which is a similarity with Complex numbers* (*Real and Image*) , then , ***Monads are composed of Real and Imaginary parts as Complex Numbers are ,*** i.e.
Objective reality contains both aspects (Real and Imaginary **,** discrete , AB , and Continuous **,** Impulses **P_A , P_B** , etc.) ***Momentum or Potential or Induced Potential.***
Change (motion) and plurality are impossible in Absolute Space [PNS] and since is composed only of Points that consist an Unmovable Space , then neither Motion nor Time exists i.e. a constant distance AB = **ds** = monad anywhere existing is motionless . The discrete magnitude **ds =[AB /n] > 0** = *the quantum* , and for infinite continuous **n , then ds** convergence to **0** . Even the smallest particle (say a photon) has mass , *the reaction to velocity change* , [15] and any Bounded Space of ds > 0 is not a mass-less particle and occupies a small Momentum which is the motion .

The Physical world is scale-variant and infinitely divisible , consisted of the finite indivisible entities ds = AB → 0 called monads and of infinite points (ds = 0) , i.e. The Euclidean and the Material Geometry.
All entities are Continuous with points and Discontinuous , *discrete* , with ds > 0 .

In PNS dt = 0 *, which is the meter of velocity changes* , so motion cannot exist at all , *ds=v.*
Since any points A,B of PNS coincide with the infinite Points , of the infinite Spaces , Anti-Spaces and Sub-Spaces **of PNS** , and since Motion may occur at all Bounded Sub-Spaces then this ***Relative motion*** is happening on the ,**e**,
-dimensional to **xx** Space and ↔ Anti-Space (the Straight line) between all points belonging to PNS and all those belonging to other Spaces .
Time exists in Relative Motion and it is the numerical order of material changes in the PNS - Space , and is not a fundamental entity as is said in Relativity. On *Monad* AB , in any Space-level , and *Primary Space*. which is = 0 ↔ AB ↔ ± ∞ exists < *a*
This Energy monad is modified as the Quanta
bounded State of energy for each one between of Energy and is represented as the Dipole of *the Infinite Spaces and Anti-Spaces* > and the energy monads in any Space-level .
[Dipole AB = Matter = monad] is the **2.. Euclidean and Non-E Geometries .** communicator of Impulses [P] *of the* Synopsis 1 :

Primary point *, which is nothing and* **has not any Position** *may be anywhere in Space , if there is any Space , therefore , the unique Primary point ,* A *, being nothing also* **in no Space** *,* **is the only Point** *and no-where, i.e. Primary Point is the only Space and from this all the others which have Position ,and because it is the only Space thus to exist point* A *, at a second point* B *somewhere else , point A must move towards point* B *, where then* A ≡ B *. Point* B *is the Primary Anti-Space which Equilibrium point* A *, and it is* [PNS] = [A ≡ B] *. The position of points in* [PNS] *creates the infinite dipole* AB *and all the quantum quantities which acquire Potential difference and an Intrinsic momentum* (± Λ) *in the three Spatial dimensions* (x , y , z) *and on the infinite points of the* (i) *Layers at these points , which exist from the other Layers of Primary Space , Anti-Space and Sub-Space , and this is because Spaces ≡ monads ≡ quaternion . Since Primary point ,* A *, is the only Space then on this point exists the Principle of Virtual Displacements ,Work* W= $\int_A^B P.\,ds=0$ *or* $[ds.(P_A + P_B)= 0]$ *, i.e. for any ds =vector>0*
Impulse $P = (P_A + P_B) = 0$ *and* $[ds.(P_A + P_B) = 0]$ *, Therefore , Each Unit* AB = ds > 0 *, exists by this Inner Impulse* (P) *where* **(** $P_A + P_B$ **)** = 0 *. This Monad , discrete , (Unit ds = Quaternion)* $\overleftrightarrow{AB}$ *is the ENTITY and* [AB – P_A , P_B] *is the LAW ≡ the Content , therefore Entities are embodied with the Laws .*

←
Entity is quaternion $\overrightarrow{AB}$ ≡ a+b.i = r.$e^{(i\varphi)}$ *, and Form* |AB| = *Energy length (the energy-quanta) of points* |A ,B| *or the wavelength , and imaginary part are the equal forces* P_A , P_B *as fields , or the medium , in monads . Line segment* AB *is not continuous unless a* ***Common Dimensional Unit*** AT >0 *or* AT = ds → AB *is accepted and thus in this way exists* TA+TB= AB *then point* T *is* ON *straight line* AB *,i.e. the whole* AB *is equal to the parts* TA ,TB *, where then issues the Property of Equality and the relation between Whole and Parts . This property in Geometry issues in all Physical levels .*

Primary Segment

is of the ***Form*** $\overleftrightarrow{AB}$ *, where* ***Form*** |AB| *,* ***Finite*** AB *and* ***Infinite*** *,∞, to Point zero*
$L_v=e^{i.(\frac{N\pi}{2})b=10^{N=-\infty}}$ *, and for* N =∞ → 0 *, where exists , the* ***Content is Atraction*** P_A↔P_B *and the* ***Repulsion*** P_A→ ←P_B *, and the* ***Quantity*** *in Real part* AB ≡ L_v *, and in Imaginary part where* **(**$P_A + P_B$**)** = 0 *, and when the* ***Quality*** **(**$P_A + P_B$**)** ≠ 0 *is a differentiation , and so on .*

Since also ***Imaginary Part*** *is always* $(P_A + P_B) = 0$ *then Form and Content are absolutely inseparable and pass from zero for all Opposites , so all Entities are embodied with the Laws , and since also valid* $(P_A + P_B) \neq 0$ *then , the* ***Zero equality is the Critical-Energy-Quantity*** {CEQ} *for any transition in Quality , a kind of Catalyst which is not changing the composition of Primary Segment , it is* the unity of opposites *and since also Work ≡ Energy involved in all levels , and Generally → is holding that ,*

In Primary Segment $\overleftrightarrow{AB} \equiv a+b.i = r.e^{(i\varphi)}$ ***exists the Contratiction and Identity ,*** *an Extrema-state of Unstable-equilibrium on the edge of nothing , or the opposites interpenetrate in Unity , or Similar charges Repel each other whereas opposite kinds attract , or a Tiny - Energy - Space , Anti-space containing Work ≡ Energy ≡ Eternal Self-Motion as Wave , forming the Material world ,* ***Apriori*** .

The Ideal is nothing else than this Material-world reflected by the human mind and translated into forms of thoughts . Since Monads are quaternion as $w = a+b.i = r.e^{(i\varphi)}$ *, composed of Real* (a) *and Imaginary part* (bi) *as Complex Numbers are , so Work , Energy , is Monad`s the most characteristic .*

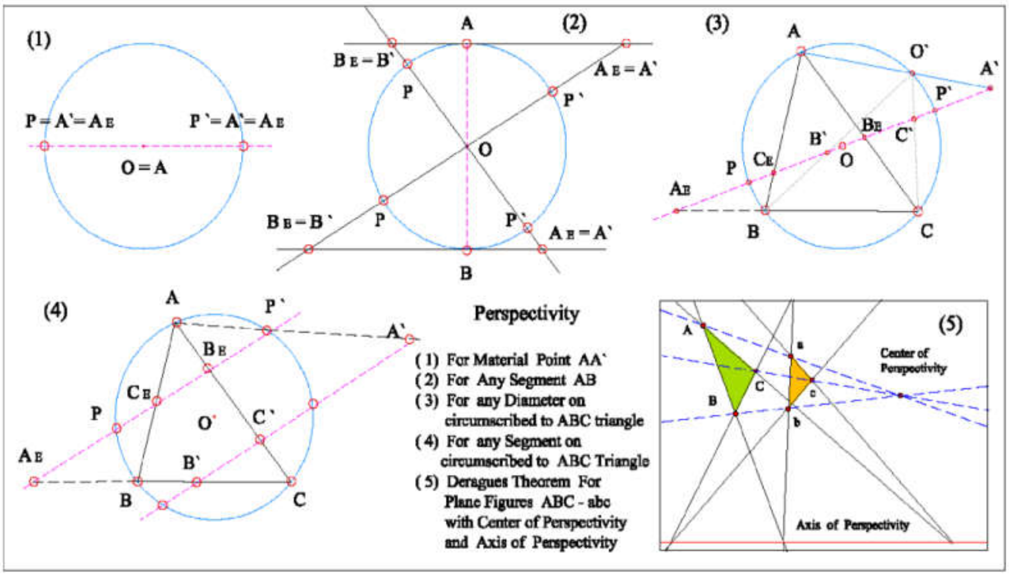


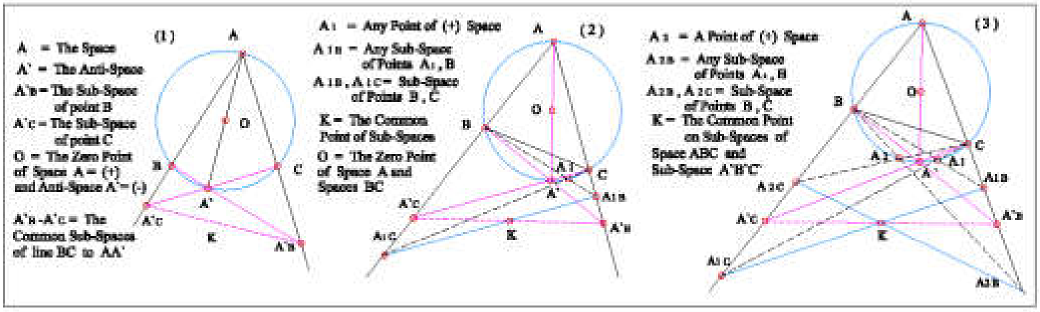
Figure.1.. Pole and Axis of Perspectivity for Points , Sectors , Planes , Volumes. ***The two Perspective Desargues triangles ABC – abc with their Axis and Center of Perspectivity***

2.1. Perspectivity :

Projective in geometry has to do with Points , Lines , Planes and Spaces embedded in Euclidean geometry as in Fig.1.
In (1) Perspective Points P , P` lie on line PP` which is monad AA` , and where O is their middle point of this material point AA`.
In (2) Perspective Points P , P` lie on the circumference of the circumscribed sphere of Plane ABO through AB axis , where O is the common circumcenter of Segment AB.
In (3) Perspective Points P , P` lie on the
Diameter of the circumscribed circle in Plane
ABC , where O is the circumcenter of triangle ABC and O` is the concurrent point on circle .
In (4) Perspective Points P , P` lie on any segment of circumscribed circle in ABC Plane , with O as center and parallel to conjugate A`B`.
In (5) Perspective Points P , P` lie on the Axis of Perspectivity of the Planes of circumscribed circles of Planes ABC , abc being centrally perspective.
Projective geometry , (***Desargues` theorem*** *) , declares that , two triangles ABC , abc are in perspective axially , if and only if they are in perspective centrally .*
We will show that , ***Perspective and Projective Geometry*** is embedded and it is an **Extrema** in Euclidean geometry.
Proof :
a.. In F1-(4) , Two points P , P′ on circumcircle of triangle ABC , form **Extrema** on line PP′. Symmetrical axis for the two points is the mid-perpendicular of PP′ which passes through the center O of the circle, *therefore* the Properties of axis PP′ are transferred on the Symmetrical axis in rapport with the center O (central symmetry) *, i.e.* the *three points of intersection* A_E , B_E , C_E *are Symmetrically placed as the other three points* A′ , B′ , C′ *on this Parallel axis .*

b.. In F1-(3) points P , P′ **are on any diameter** of the circumcircle , and then line PP′ coincides with the parallel axis , the points A′ , B′ , C′ are Symmetric in rapport with center O and *the Perspective lines AA′ , BB′ , CC′ are concurrent in a point O′ situated on the circle .*
*When in F1-(5) , a pair of lines of the two triangles (ABC , abc,) are parallel , then extrema case is when their point of intersection recedes to infinity ,and axis PP′ passes through the circumcenters of the two triangles , (Maxima) and is not needed " **to complete** " the circumcenter of triangle .*
From above is shown that *Perspectivity* exists between any triangle ABC , a line PP′ and a center O , where then exists Extrema for each Point , Line , Plane , Space etc. i.e.

Perspectivity on a Plane is transferred on lines and from lines to Points . This is the compact logic into Euclidean geometry ,
Perspective lines of two Symmetric triangles in a circle are concurrent in a point , on the diameters and through the vertices of the corresponding triangles .
c.. When all pairs of lines of two triangles are parallel , the equal triangles , then points of intersection recede to infinity , and axis PP` passes through the circumcenters of the two triangles (The Extrema case).
d.. When the second triangle is a point P then axis PP′ passes through the Projective and Perspective - Geometry is an Extrema in Euclidean–Geometry in all levels without controversy or contradiction .
Mathematical interpretation and all the relative Philosophical reflections based on the Non - Euclid geometry theories , must be properly revised and resettled in the truth one.
For conceiving alterations from Point to sectors discrete , lines , plane and volume is needed Extrema knowledge where there happen the inner transformations on geometry and the



2.2. The Extrema Euclidean Geometry:
1.. In Figure .2. Extrema of a point A is point A\` on Straight line AA\` and the middle point of segment AA\` is point O with equal distance OA = OA\` . From point O is drawn the only one circle (O, OA=OB) on which exist infinite points forming any triangle ABC in the circle of this diameter AA′. Point A represents the Space and point A\` the Opposite Anti-space.
In E-geometry the two points equilibrium because of ***equal distances*** OA **, OA\`** from midpoint O while in Material-Geometry equilibrium because of ***equal Forces*** $\mathbf{P_A}$ **,** $\mathbf{P_{A`}}$ at end points A , A\` , from midpoint O . i.e. it is
Dipole =[⊕⊖]=□=AB .

Is shown also the relation between point A\` which is the Anti-space , with the three points A,B,C representing the Space-Plane . Lines CA′, BA′ produced , intersect lines AB , AC at points $A`_C$, $A`_B$ respectively . Points $A`_C$, $A`_B$ represent the Sub-space of , Space , Anti space A↔A\`.
A1 is any point on the circle between the points B , A′CA1 , BA1 produced intersect lines AB , AC at points A_{1C} , A_{1B} respectively. **Show** that lines A_{1C} , A_{1B} are concurrent at the circumcenter K Considering both angles < $A`_CBA`_B$=$A`_CCA`_B$= 90° then lines $BA`_C$ , $CA`_B$ produced meet lines $AA`_C$, $AA`_B$ at points A_{1C} , A_{1B} such that line A_{1C} A_{1B} passes through point K
(*the common to* A_{1C} A_{1B} , $A`_CA`_B$ *segments*) and when angle <BAC = 0 as extrema case then point K , coincides with Anti-space point A\` which are both on the circle ,
i.e. From all contrary cases , In an angle <
The circle (O , OA = OB = OC) is the inscribed in triangle $K_AK_BK_C$ **and the circumscribed on triangle** ABC **.**
In all Plane levels of Euclidean Geometry , the Space points A , B , C , the Anti-Space points [A\`,B\`,C\`]≡ [$\mathbf{A_E}$, $\mathbf{B_E}$, $\mathbf{C_E}$**] *, and Sub-Space points*** $\mathbf{K_A}$, $\mathbf{K_B}$, $\mathbf{K_C}$ ***lie on the Circumscribed circle and Circumscribed to* ABC *triangle and it is the Extrema of it , to its vertices . This coexistence of the three Spaces in One is the main property of Spaces and into this Mechanism Stabilizer is the Work ≡ Energy as Glue-Bond between them .*** [58] ***Theorem :*** *On any triangle* **ABC** *and the circumcircle exists one inscribed triangle* $A_EB_EC_E$ *and another one circumscribed* ***Extrema*** **triangle** $K_AK_BK_C$ *such that the* ***Six*** *points of intersection of the six pairs of* ***triple lines*** *are collinear →* (6+6+6) = 18 . Fig.3 –(3) The six-triple points-line [STPL] is line → of Points D_A ,D_B ,D_C - P_A ,P_B P,C where ,
Triangle **ABC** → is the Space Triangle.

Triangle **B_EC_E** → is the Anti-Space.
Triangle **K_BK_C** → is the Sub-Space Plane.

Proof : Fig.2 - Fig.3. (3) - Fig.4

Let ABC be any triangle (The Space) , the K_A , K_B , K_C are the points of intersection of tangents at A,B,C points of circumcircle (The Sub-Space) , A_E , B_E, C_E be the points of intersection of lines AK_A , BK_B ,CK_C and the circumcircle (The Anti-space) respectively .

1.. When points A_1 , A coincide , then internal lines CB_1 , BC_1 coincide with sides CA , BA , so line K_AA is constant . Since point A_E is on Extrema line AK_A then lines C_EB , B_EC concurrent on line AK_A . The same for tangent lines K_AK_B , K_AK_C of angle $< K_BK_AK_C$.

2.. When points A_1 , B coincide , then internal lines CA_1 , AC_1 coincide with sides CB , AB , so line K_BB is constant . Since point B_E is on Extrema line BK_B then lines A_EC , C_EA concurrent on line BK_B . The same for tangent lines K_BK_C ,
K_BK_A of angle $< K_C K_B K_A$.

3.. When points A_1 , C coincide , then internal lines AA_1, BA_1 coincide with sides CA , CB , so line K_CC is constant . Since point C_E is on Extrema line CK_C then lines B_EA , A_EB concurrent on line CK_C . The same for tangent lines $K_C K_A$, $K_C K_B$ of angle $< K_A K_C K_B$, i.e.

Triangles* ABC , $A_EB_EC_E$, $K_AK_BK_C$ *are Perspective between them , and consequently between the Spaces.

Since Triangles ABC , $A_EB_EC_E$ are Perspective between them , therefore the pairs of Perspective lines [AA_E, BC_E,CB_E] , [BB_E , CA_E , AC_E] , [CC_E , AB_E , BA_E] are concurrent in points P_A , P_B , P_C , respectively .

Since Triangles ABC , K_A K_B K_C are Perspective between them , therefore the pairs of Perspective lines [K_BA , CB , C_EB_E] ,
[K_AB , AC , A_EC_E] , [K_BC , BA , B_EA_E] , are concurrent in points D_A , D_B , D_C respectively .

Since lines (K_AK_B , K_B K_C , K_CK_A) are Extrema (tangents to circumcircle) for both triangles ABC and $A_EB_EC_E$, of sides
(BC , B_EC_E) , (AB , A_EB_E) , (AC , A_EC_E) , then , the points of intersection of these lines lie on the same line . i.e.

***This compact logic of the points* [A , B , C] ,**
[A_E, B_E, C_E] , [K_A, K_B , K_C] *when is applied on the three lines* (K_AK_B , K_B K_C , K_CK_A)
then the* SIX *pairs of the corresponding lines which extended are concurrent at points P_A , P_B , P_C ***for the triple pairs of lines* (*Pascal`s Perspectivity of points in Euclidean geometry*) , [AA_E, BB_E, CC_E] ,**
[BB_E, CA_E, AC_E] , [CC_E, AB_E, BA_E] *and at Points*
D_A , D_B , D_C ***for the triple pairs of***
lines* [K_B K_C , BC , B_EC_E] , [K_AK_C , AC ,A_EC_E] and [K_AK_B , AB , A_EB_E] , (*Desargues`s Perspectivity of points in Euclidean geometry*) *and all the 18 common points lie on a straight line the→ STPL Mechanism .

As proved **,** Straight line AA_E is continuous in ,ds, with ds = 0 as points of filling , and also discontinuous (*discrete*) with the dimensional Units , ds ≠ 0 , defining the Space , Anti-space at A ,A_E points and Sub-space at K_A , where , *ds = quantum* = AA_E/ n , (where n = 1,2,3 → ∞ , = [a+b.i] /n = complex number *and Infinitively divisible which is keeping the conservation of Properties at End Points* A , A_E) as filling , ***and continuous with points as filling*** *(and for n = ∞ then ds = 0 i.e. the ∞ Positions of points in ds)* . On line AA_E exists Euler-Savary mechanism for Couple-Curves.

2.3. Remarks on The Physical meaning of the Geometrical Properties.

The [STPL]≡Six Triple Points Line Mechanism.

The Geometrical mould on Physical world :

1.. [STPL] is a ***Geometrical Mechanism*** that produces and composite all opposite Space Points from Spaces (The three characteristic points A-B-C forming a Plane) , Anti-Spaces
(The corresponding points $A_EB_EC_E$ of opposite direction through the Zero space) and the Sub-Spaces (The Zero Plane points K_A , K_B , K_C is similar to Positive axis which passes from Zero in order to pass to the Negative axis) in a Common Circle , *Sub-Space* , line or a cylinder .

2.. Points A,B,C and lines AB,AC,BC of ***Space , communicate with*** the Extrema corresponding A_EB_E , A_EC_E , B_EC_E , of ***Anti-Space*** , separately or together with bands of three lines at points P_A , P_B , P_C , and with bands of four lines at points D_A ,D_B ,D_C ***on common circumscribed circle*** (O , OA) , ***consisting the Sub-Space***. [17]

3.. If any monad AB (*quaternion or Vector*) , [s , v̅.□i] , *all or parts of it* , somewhere exists at points A,B,C or at segments AA_E ,BB_E ,CC_E then [STPL] line or lines , is the Geometrical expression of the Action of the External triangle , K_A K_B K_C , *the tangents as extrema is the Subspace* , on the two Extreme triangles
ABC and A_E , B_E, C_E (*of Space Anti-Space*) creating 1,3,5, spin , *the minimum Energy - Quanta* .(this is the How Opposites combine and produce the Material-Neutral) . [29]

When the ***monad*** (quaternion with real part = s = 2r and Imaginary part v̅ =□i = Λ̅ = Ω = m.v.r) is in the recovery equilibrium
(*a surface of a cylinder with 2r diameter*) , and because velocity vector is on the circumference , then the two quaternion elements identify with points ,A,B,C (of the extreme triangles ABC of Space ABC) and Imaginary part with points A_E

, B_E, C_E (of the extreme triangles $A_EB_EC_E$ (of Anti-Space) , on the same circumference of the prior formulation and are rotated with the same angular velocity vector $\overline{\mathbf{w}} = 2\pi f$. The inversely directionally is the rotated Energy $\pm\overline{\Lambda}$ and equilibrium into the common circle , so Spaces and Anti-Spaces meet in this circle which is the common Sub-space . Extreme Spaces (the Extreme triangles ABC) meet Anti-Spaces (the Extreme tangential triangles $A_EB_EC_E$) , through the only Gateway which is the center O of the Plane Geometrical Formulation Mechanism (mould) of the [STPL] line . [43]

Since the origin of Space [S] *becomes, through the Principle of Virtual Displacements as , W=*$\int_A^B Pds$ = *0 from Primary Point* A *which is the Space , to* A_E *which is the Anti-space as the Inner distance ,ds, of Space and Anti-Space in all Layers then , Distance* $ds = AA_E$ *is the Work embedded in monads and it is what is vibrated .*
Since also Work of the Inner Impulse distance of Space and Anti-Space is embedded in all material points of universe , stationary points *, a Torsional Oscillation* $\overline{\Lambda}$ *in STPL mechanism happens and thus a Natural Wave-Frequency* $\mathbf{f_m} = \mathbf{w}/2\boldsymbol{\pi}$ *is embedded in Material-Geometry , from which exist the Euler-Savary equations with the rotating and Rolling curves , and thus become the figures of Conchoide to Spirals and all the others .* [58]

Point, which is nothing and has not any Position may be anywhere in Space , therefore , the Primary point A , being nothing also in no Space , is the only Point and nowhere, i.e. Primary Point is the only Space and from this all the others which have Position , therefore it is the only Space and so to exist point A at a second point B somewhere else , point A must move towards point B , where then A ≡ B. Point B is the Primary Anti-Space which
Equilibrium point A , [PNS] = [A ≡ B] .
The position of points in [PNS] creates the infinite dipole and all quantum quantities which acquire Potential difference and an Intrinsic moment ± Λ in the three Spatial dimensions (x , y , z) and on the infinite points of the (i) Layers at these points , which exist from the other Layers of Primary Space , Anti-Space and Sub-Space , and this is because Spaces = monads = quaternion [9] . Again , since Primary point A , is the only Space then on this point exists the Principle of Virtual Displacements as ,

$$W = \int_A^B P.ds = 0 \text{ or } [ds .(P_A + P_B) = 0] ,$$

All points may exist with P = 0 → (PNS) and also with P ≠ 0 → (Spaces) because , ($P_A + P_B = 0$) for all points in Spaces and Anti –Spaces , therefore [PNS] is self -created , and because at each point may exist also with P ≠ 0 , then [PNS] is a (perfectly Homogenous , Isotropic and Elastic Medium ,in spatial and Temporal domain) Field with infinite points which have a ± Charge with P = 0 → P = Λ → ∞ .
Work (W) is quantized on material-points as EM wave and spin $\pm(\overline{p})$ and from this , equilibrium and quantized angular momentum $\overline{\Lambda}$ which is independently of time and is capable of forming the Wave nature of Spaces , following the Boolean logic and distorting momentum $\overline{p} = \overline{\Lambda}$, as energy , on the intrinsic orientation position of points , on all points of the microscopic and macroscopic homogeneity .
Since also in common circle rotational velocity ,$\overline{w}$, and momentum , $\overline{\Lambda}$, are constants *, and because of these the constant velocity ,**c**, is created* ,so thus it consist a Pure quaternion which is the cause of their Inner motion , (*This is the Electromagnetic wave which produces Spin*) and of their Outer Spin (*This is the screw helically Kinetic Energy wave Motion conjugation)* . Conjugation equation of the two gives ,

$$(\partial/\partial t , \overline{w}) © (0 , \Lambda) = (- \Lambda , wx\Lambda) =$$
$$(-\overline{H}x\overline{P}, \square x\overline{\Lambda}) = [\lambda , \square \ x \ \overline{\Lambda}] . [13-15] .$$

3.. The Material Geometry and Properties . All above Geometric logic is simultaneously presented on Space , Anti-space and on the deep relation of the Material-Geometry and Physics , because by Considering → point A as the positive Space = ⊕ , point A_E as the negative Anti-Space = ⊖ , and point K_A as the Neutral =
Space then , in Fig.5 ,

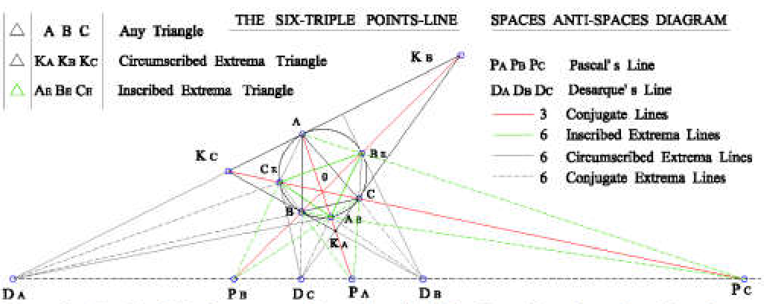


Figure.4.. *The Six , Triple Concurrency Points , Line .* [STPL] → D_A , D_B , D_C - P_A , P_B , P_C

This Property of links ,constitutes the Instaneous rotation of , Plane Space , Anti-space , [For point A is the Rotation of Triangles OAD_A , $OA A_E$ with velocity , $\overline{v}$, on the circumference of circle (O,OA)] *with Instaneous centers of rotation* D_A , P_A *on STPL Line , where then equilibrium happens on* A K_A *straight line . Simultaneously Euler - Savary equation* *relates three directed quantities lying on the path normal* AK_A and reduces to having K_AA_E, K_AP_A *always laid off in the same sense along the line* AK_A, *and also the converse of Positions since inflection circle* (O,OA) *is the location of couples points whose curves have an infinite radius of curvature as in Figure 5. where angle* < $AOA_E = 180^\circ$. *Euler-Savary equation gives the radius and the center of curvature of this coupler curve between the Instaneous Rotation of , Space and Anti-space .*

In Figure. 5-6-7 , ***is shown the Lorentz factor*** **$\gamma \equiv sec.\varphi$** ***, becoming from*** **STPL mechanism and** ***related to All known Particles , following the Conchoide of Nicomedes to COSC .*** [58]

Gravity force is exerted on breakages [± $(\bar{w}.r)^2$ = Material points = Dipole of the two ± quantized energy-spaces $(\bar{w}.r)^2$] as velocity vector , $\bar{c}$, which is then decomposed into two reverse velocities following the cycloidal motion , and consisting the intrinsic Stationary Electro-magnetic Wave of gravity, and which is binding points of this Homogenous-Isotropic, Rest and mass-less nature Field.

The total dispersion Rotating energy of dipolesis [± $\bar{\Lambda}$ $]^2 = [p.c]^2 + [m_o.c^2]^2$, which is the known relativistic energy-momentum for r all motions, and Gravity as force [□i], while the clashed with the constant velocity ,$\bar{c}$, consist the Dark matter [± $\bar{c}$.s] and the Dark energy [$\bar{c}$.□i] , or from → Breakages

[± s^2 = ± $(wr)^2$] and [□i = $2(wr)^2$] where then become Waves { *Distance* $ds=AA_E$ *is the Work embedded in monads and it is what is vibrated }* with Vibrating equations of motion becoming as

A → Particles, with Inherent Vibration ,

B → Gravity-field-energy, without Vibration

equation of Lorentz transformation equations. C → Dark-matter-energy constituents as,It has been shown [16] that Projective and Perspective geometry are Extrema in Euclidean geometry into [STPL] line , their boundaries becoming from common Space and Anti-space. Energy , *Motion*, follows this Euclidean moulds, because this Proposition , *Principl*e, belongs to geometry, and not to Energy which is only motion.

In [33-36] Un-clashed Fragments through center O, consist the Medium-Field Material-Fragment →[± s^2] = [MFMF] as base

A.. [±$\bar{v}.s^2$] → Fermions and [$\bar{v}$.□i] → Bosons,

B.. [± s^2] → [MFMF] Field, and the binder , Field is [□i] → Gravity force, C.. [±$\bar{c}.s^2$] → Dark matter , and the binder Gravity force [□i] , [$\bar{c}$.□i] → The Expanding Dark energy.

From above is seen that in , **A** ***, and ,*** **C** ***, case Energy as velocity ,*** **$\bar{v}$** ***, exists in the Discrete monads ,*** **$\pm\bar{v}.s^2$** ***and*** **$\pm\bar{c}.s^2$. B** ***case , is the transportation of Energy from Chaos to Material points*** **[$+s^2 \leftrightarrow -s^2$]** ***as in page*** **30 [B].**

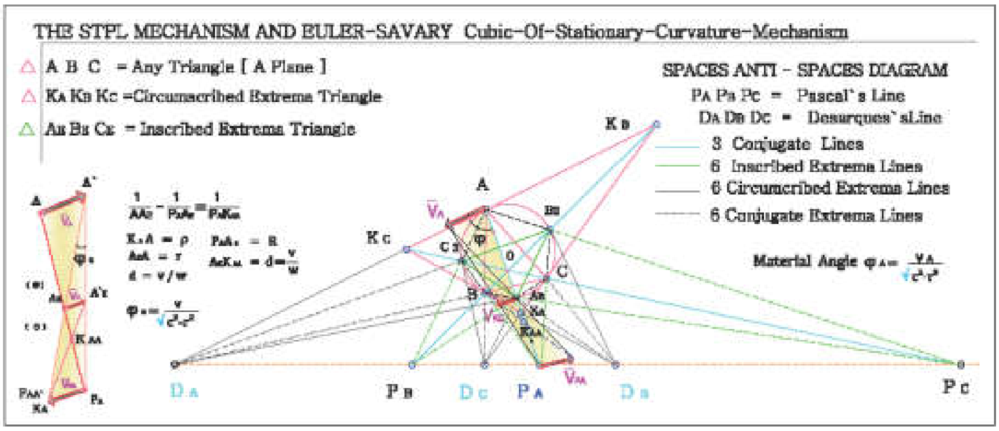


Figure.5.. ABC is any triangle (The Space) , KAKBKC triangle is the (The Sub-Space) , AEBECE triangle is (The Anti-space) respectively . The Instaneous Pole **P** ≡AE of rotation is off the circle on AAEaxis .Inscribed to ABC circle is Common circle of STPL-{DA-PA} mechanism *The Reference System* {DA- PA}≡[R] (x',y',z', t') *moves with velocity* ,$\bar{v}$, *parallel to* , x-x', *axis with respect to the fixed and Absolute System* {DA- O} ≡ [S](x,y,z, t) **.** *The* Space point , A , moving on (p) curve , and Anti-Space point AE moving on (ε) curve are rolling on the same Sub-space circle (O,OA) ≡ (O,OAE) which is the common cave-circles of Material Geometry in STPL-{DA-PA} mechanism .

3.1. The Instaneous Pole P ≡ A_E of rotation, on the Inflection-circle of Plane AD_AA_E of STPL

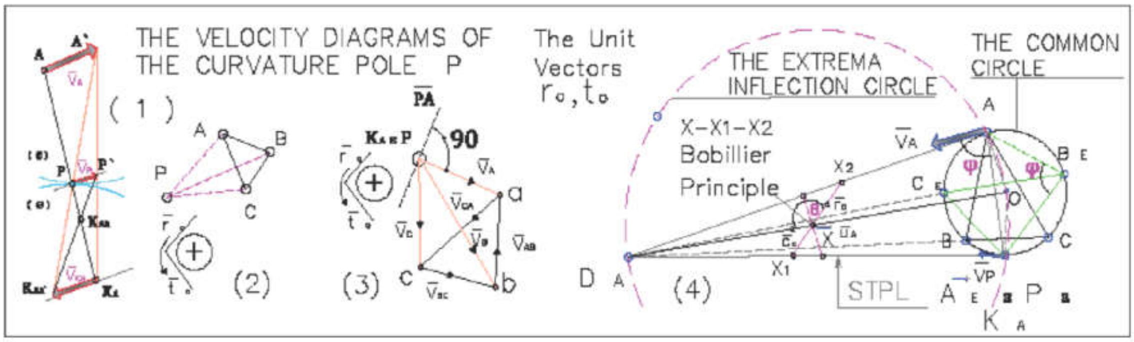


Figure.6.. ABC ,is any triangle (The Space) , $K_AK_BK_C$ triangle is the (Sub-Space) , $A_EB_EC_E$ triangle is the(Anti-space) respectively. The Instaneous Pole **P** of rotation coincides with Anti-space point A_E on the circumscribed to ABC Circle .

The Velocity diagrams for the Instaneous Pole **P** of rotation in STPL≡ [O,ABC]-{ D_A-P_A} mechanism , on the inflection circle of the Plane points A , B , C .

In (1) point K_{AA} is the velocity instaneous center for point A in S_o system .

In (2) point P is the Pole of rotation for points A , B , C .

In (3) Figure is the Velocity Diagram P–a ,b, c for points A , B , C

In (4) When STPL is Tangential to (O,OA) circle then the two circles , *The common-circle and Inflection circle* , cut on AP chord which is common to Velocities , and the Accelerations of points A , P , coincide with D_A , P_A Desargues and Pascal`s points .

On AD_AA_E , Material lines X_1XX_2 , formulate all referred curves .

Any rotation in three dimensions can be represented as a combination of a vector $\mathbf{\bar{v}_A}$ and of a scalar angle ,φ, on AA_E≡AP axis which is the *Euler rotation theorem-axis* . [58]

3.2. The Angular Momentum of any where then for instaneous velocity v = w.r , then **Material point in STPL mechanism .** L = m(w.r) . l i.e. Angular momentum is equal From Physics momentum p = m.v = m $\frac{ds}{dt}$ …(1) to the followings →

where → mass = the reaction to the change of velocity → |v| = the instant velocity equal to ds/dt which is the change of displacement ds , where ds = l , is Dipole =|[⊕⊖]|=□=l=AB . [40] Angular Momentum L = l x p = |l|.|p|.sin φ ..(2) where φ = Angle subtended between direction of l and p . [41] 1.. To the reaction ,m, of the change of position vector ,l, through material point axis AB .

2.. To the Intrinsic angular velocity ,w, of the material Point as a cave.

3.. To circular orbit of radius ,r, of material point. 4.. The length |l| of the position vector which is the wavelength λ = 4π.r of the material point .

l = a position vector . Differentiating (2) Since any Monad , (Unit) $\overrightarrow{AB}$= L , is the ENTITY pxp and [A,B – P_A , P_B] is the $\frac{dL}{dt} = \frac{d\,l}{dt}$ LAW , $\frac{dp}{dt}$ so Entities are then x p + l x = vxp + l x F = embodied with the Laws. + l x F = 0/m + l x F = l x F = J $\overline{m}$. ā ……(3) , Since Entity is quaternion , and law |AB| = where J = moment of Inertia $\overrightarrow{AB}$ a= acceleration.

Since **p = m** x **v** , and which is a Torque acting on length = the Real part which is the Space of the particle about its axis through l , or points A,B then imaginary part (i)are the forces P_A,P_B or the fields in AB .

$\frac{dL}{dt}$ = l x F → is a Torque also , i.e. By definition i =√-m.1 and(-m1)²= -1m i.e.

It is the Linear momentum . [Energy]² = - [Space] = Anti-space and since ***Remark*** : $\frac{dL}{dt}$ = l x F = Torque → which also exists Λ x Λ = - (- m.1) ² = ± Λ.□i , the basic suggests that , equation (3) is the ***Extrema case*** equation of quaternion becomes [-(ΛxΛ)/m ± ***between , the Linear and Angular Momentum*** , Λx□i] = [λ , ± Λx□i] i.e. wavelength λ = - (ΛxΛ)/m where m = a constnant depending on the reactions to the present or other conditions . Applying this in energy cavities then λ =

$$= e^{-i[(\frac{\pi}{2})b]^2} = e^{-i[(\frac{2\pi}{2}).b]} = e^{-i[(\pi).b} \rightarrow \text{i.e.}$$

The Massive mechanism Diffraction and the Energy mechanism Diffraction , *The Quanta* , are Interchangable as, $e{-}i.(1{,}78.10{-}7)^2 = e{-}i.(3{,}56.10\ ^{-}14)$ and for Relativity massive Energy (ΛxΛ) =
(- m.i) x (- m.i) = m $(i)^2$ = - m .(v̅)^2 = - m..$\bar{v}^2$, where imaginary part , i = v̅ , i.e.

3.3. The Absolute and Relative Motion . *The Space aquires energy as velocity .*

Applying quaternion equation [-□Λ ,□xΛ] = 0 for point ,O, and constant velocity , c̅, then . $[-\nabla\Lambda , \nabla x\Lambda] = 0$

where $[-\nabla c] \perp [\nabla xc]$ meaning that it is a mechanism that instantly transports breakage masses in two directions dynamically and perpendicularly to all Inertial frames Layers .

From Velocity-Energy vector are produced the three breakages $[\pm s^2 = \pm (wr)^2]$ and $[\nabla i = 2(wr)^2]$ and from them Fermion and Bosons . [26]

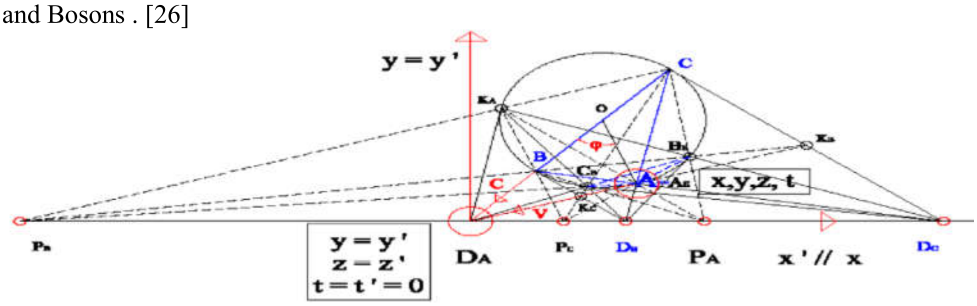


Figure.7.. ABC is any Right-angled triangle at A (The Space) , $K_AK_BK_C$ triangle is the (Sub-Space) , $A_EB_EC_E$ triangle is the(Anti-space) respectively. The Instaneous Pole **P** of rotation is off the Circle of diameter BC . The Poles of rotation lie on $\{D_A\text{- }P_A\}$ Reference system.

Reference System $\{D_A\text{- }P_A\}$≡[R] (x',y',z', t') *moves with velocity ,v̅, parallel to ,* x-x', *axis with respect to the fixed and Absolute System* $\{D_A\text{- }O\}$ ≡ [S](x,y,z, t) .

The Geometrical expression of Lorentz factor ,γ, is as **sec φ** = **γ** = OD_A: AD_A =
± 1 / [√1 – (v/c)²] and which is the Conchoide of Nicomedes , { s = a + b. sec φ } , and which acquires the material Angle $\varphi = \frac{v}{\sqrt{c^2-r^2}}$

The Relative Motion

Because Properties In and On [STPL] line , are relative to the only one Equilibrium and Absolute system ± Λ = r.mv̅ = r.m.w̅.r = mr².w̅, so exists that what is called Relativity. As Absolute System let it be [S] ≡ {DA-O} ≡ STPL mechanism , and as the Relative (Reference,
Affine) System, [R] ≡ {DA-PA} . Fig- 7 their intrinsic rolling circles . In F-7 , this relation is Geometrically expressed as →

sec φ = O DA : A DA = γ =
± 1/ [√1–(v/c) ²] = c / [√c² – v²] , and it is a geometrical Cycloid property equal to Lorentz`s ,γ, factor. Newton`s laws are true into Reference System [R] ≡ {DA-PA} by ,
Considering {DA-O},(x,y,z, t) , as the fixed
The Relative motion [S]≡{DA-O} , [R]≡{DA-PA} frame [S] of the coordinate system in the
of the two above Systems :
It was shown , that in {DA-O},(x,y,z, t) , System c̅,v̅, vectors are isochrones i.e. period T = L/V = 2πR/V = 2π/[c/*rc*] = 2π/[v/*rc*] → c/rc= v/rv → c.rv= v. rc ,where rv , rc are the radius of Gravity cave (d=2r) and point A(x,y,z) is fixed on circle (O,OA) which is rotating with a velocity v̅= w̅r and of angular velocity w̅ =2π/T = 2πf where period of rotation ,T, is also constant .
Since acceleration for a quaternion z = (s +
v̅.□i) is a= [d²z/dt²] = (ds/dt.v̅.□i) + s.d(v̅.□i)/dt
= 0 + s.d(wr)/dt = 0 + 0 , and this because w̅ = constant for both , therefore , velocity v̅ = constant also , i.e. →
Centrifugal velocity of Absolute system [S] is any constant , **c̅** , and this because angular velocity ,**w̅**, is constant also and thus , is not needed to accept apriori this constancy which is → their wavelengths are a Stationary wave in cycloid ← following Lorentz`s factor ,γ, then this following , happens also to all frames which make this motion , and so issues
{DA-O} = γ.{DA-A} …….(2-0)

On this Relative system DA(x', y'=y, z' = z , t`) are conveyed , the Breakages [± $(wr)^2$, $2(wr)^2$]
The Relative motion [**S**]≡{D_A-O} , [**R**]≡{D_A-P_A} frame [**S**] of the coordinate system in the of the two above Systems :
It was shown , that in {DA-O},(x,y,z, t) , System $\overline{c}$,$\overline{v}$, vectors are isochrones i.e. period T = L/V = 2πR/V = 2π/[c/rc] = 2π/[v/rc] → c/rc= v/rv → c.rv= v. rc ,where rv , rc are the radius of Gravity cave (d=2r) and point A(x,y,z) is fixed on circle (O,OA) which is rotating with a velocity $\overline{v}$= $\overline{w}$r and of angular velocity $\overline{w}$ =2π/T = 2πf where period of rotation ,T, is also constant .

Since acceleration for a quaternion $z = (s + \overline{v}.\nabla i)$ is a= $[d^2z/dt^2] = (ds/dt.\overline{v}.\nabla i) + s.d(\overline{v}.\nabla i)/dt$

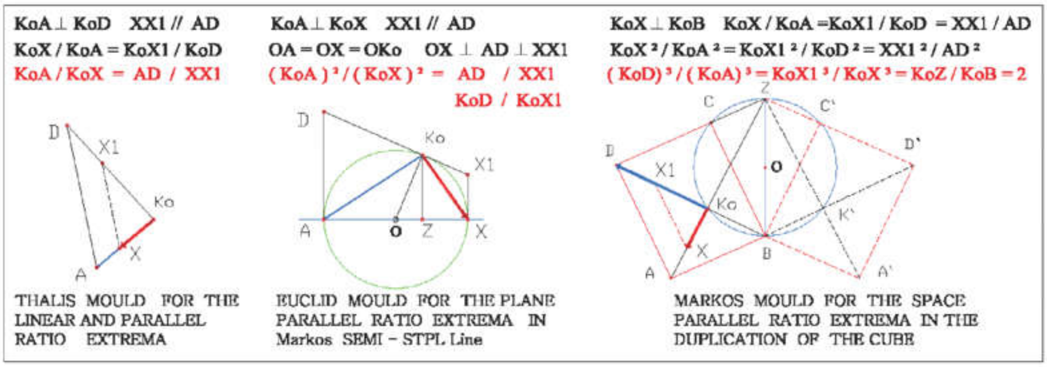
= 0 + s.d(wr)/dt = 0 + 0 , and this because $\overline{w}$ = constant for both , therefore , velocity $\overline{v}$ = constant also , i.e. → Centrifugal velocity of Absolute system [S] is any constant , $\overline{c}$, and this because angular velocity , $\overline{w}$, is constant also and thus , is not needed to accept apriori this constancy which is → their wavelengths are a Stationary wave in cycloid ← following Lorentz`s factor ,γ, then this following , happens also to all frames which make this motion , and so issues
{DA-O} = γ.{DA-A} …....(2-0)
On this Relative system DA(x', y'=y, z' = z , t`) are conveyed , the Breakages [± $(wr)^2$, $2(wr)^2$]
of velocity $\overline{c}$ = 0 → $\overline{v}$ → ∞ on circle of (O,OA) circle after the colliding with the **(O,OA)** ***to exist in frame*** , so rotating velocity $\overline{v}$ = $\overline{w}$.r of the [S] system , automatically *is defined the conversion factor* and are the fundamental particles , Fermions *t = time* , between the conventional time units (second) and length units (meter = A.D_A) or as $\overline{c}$. r_v = $\overline{v}$. r_c , → $\overline{c}$ (v)(T/2π) = $\overline{v}$(c)(T/2π) → $\overline{c}$(v)/w = $\overline{v}$(c)/w which is happening with the same ,**w**, without any restrictions , in contradiction to General Relativity which is an axiom apriori.

This is the why conversion factor , t = time, has not any essence in all universe , but it is a meter of changes only .
Because [STPL] line of the fixed frame is becoming from this system [S] , then this relative frame [R} is common to the fixed one (common D_A) and let it be [R](x',y',z', t') .
From figure Fig-7 *,* sinφ = ($\overline{v}$/$\overline{c}$) *meaning that the Relative system* , [R](x',y',z', t') *, (the Affine Frame) is the projection of Absolute Frame* [S] ≡ {D_A-O} - (x,y,z, t) *where exists as Simultaneity for all motions , i.e.*
[R] ≡ {D_A-A} ≡ [(x',y',z', t')] ,
[S] ≡ {D_A-O} ≡ (x,y,z, t) = [R] .γ ≡
(x',y',z', t')
Considering point D_A as the common center and [STPL] as the x-x axis of the two systems , then becomes D_A (x ,y=y ', z = z', t) and for all linear systems D_A (x', y'= y, z '= z , t') respectively .
This specific state of constancy , i.e., the Centrifugal velocity of Absolute system [S] to be a constant ,$\overline{c}$, and the rectilinear motion with respect to one another , defines the natural Inertial frames , and because of uniformity of Space and motion , therefore occupy the same meter of their changes , (i.e. the Time) .
Since also points O,A remove to point D_A isochrones by their intrinsic property motion , and Bosons, or by escaping consisting the Rest Field and Gravity , or Dark matter and Dark Energy , as analytically is shown . [39]
Remarks :
a.. Material point A ≡ ± |$(\overline{w}.r)^2$| of the Fixed
System {D_A-O} travels with velocity $\overline{v}$ at point
D_A , so geometrical distance A.D_A in the Relative System [R] ≡ {D_A- P_A} is A.D_A = x'+ $\overline{v}$t', and because of the isochrones motion in the Fixed System [S] ≡ {D_A-O}, it is holding , x = (x'+ $\overline{v}$.t').γ or x = (x'+ $\overline{v}$.t')γ = [x'+ $\overline{v}$.t'] /
[$\sqrt{1-(v/c)^2}$] …. (2a) Inversely, by using (2a) , where [S]≡{ D_A-A}≡
{D_A-O}/ γ , then if Material point A of the Fixed System{ D_A-O} travels with velocity $\overline{v}$ at point D_A , the geometrical distance A D_A in the Fixed System [S]≡ {D_A-O} is → A.D_A = x - $\overline{v}$.t and in the Relative System [R]≡{ D_A- P_A} it is → x' = (x-vt).γ= [x-vt] : [$\sqrt{1-(v/c)^2}$] …...(2b)

3.4. The *Quantization of E-Geometry* *and its moulds .*

It was shown in [58] that common-circle of radius , r_c , is the common source of vibration excitation for the Space , Anti-space , considered as rotating with constant angular velocity $\overline{w}$. The same also on all lines joining the Space , with Anti-space points , and the STPL line , and Particles acquire the Inherent Vibration ,
This vibration is the configuration of Conchoide of Nicomedes which is connecting the Glue-bond of the Spaces , and Generally the changes on axis, from Instaneous circle of rotation of the Plane Space , and Anti-space **AA_E** ***through ,*** **K_A** ***, Neutral point*** **of the STPL mechanism.**

3.4.1 The Quantization Meter – Moulds .



(1) (2) (3)

Figure.8. The *Thales , Euclid , Markos Mould , for the* ***Linear – Plane - Space*** , *Extrema Ratio , Meters .*

In (1) is the ***Linear - Ratio*** where , length K_oA *analogous* to monad K_oX is equal to AD / XX_1 following the Euclid`s parallels .
In (2) is the ***Squared - Ratio*** where , length K_oA *squared* to monad K_oX squared is equal to linear ratio AD / XX_1 following the Euclid`s parallels .
In (3) is the ***Cube - Ratio*** where , length K_oA *cub* to monad K_oX cube is equal to linear ratio K_oZ / K_oB following the Euclid`s parallels

Quantization of E-geometry is the way of Points to become , *discrete* , as → (Segments , Anti-segments = Monads , Anti-monads) , (Segments ,Parallel-segments = Equal monads) , (Equal Segments and Perpendicular-segments ≡ The Plane Vectors) , (The Un-equal Segments twice-Perpendicular-segments ≡ The Space Vectors = Quaternion) . [15] Monads and Segments being quaternion occupy Massive and Energy magnitudes called Meters . Since points A , B , C (of the extreme triangles ABC which denote the Space ABC) are in the recovery equilibrium with points A_E , B_E , C_E , (of the extreme triangles A_E B_E C_E which denote the Anti-Space) and meet also in the same common circle which is the Common Sub-space , therefore Energy between the two Spaces passes through Sub-space from Extreme Spaces (Extreme triangles ABC and Extrema Anti-triangle A_E B_E C_E in the Sub-triangle K_A K_B K_C meet in this circle which is the common to all spaces .

e. common-circle of radius , r_c , is the common source of vibration excitation for the Space , Anti-space , considered as rotating with constant angular velocity **,w**, . Since Space , Anti-space are on the same circle then their relative motion is the , Rolling of Space ABC on Anti-space A_E B_E C_E and since also this relative motion is applied on STPL line , then D_A , P_A , points are the corresponding linear links of vibrations and Poles of rotation . [58]
Anti-segments = Monads , Anti-monads) , (Segments ,Parallel-segments = Equal monads) , (Equal Segments and Perpendicular-segments ≡ The Plane Vectors) , (The Un-equal Segments twice-Perpendicular-segments ≡ The Space Vectors = Quaternion) . [1]

3.5. The Deduction of *Projective- Geometry* And *Perspectivity* in E-Geometry and further in *Material-Geometry*

Perspectivity and Projectivity of Points :
A.. For One point A perspective point A`, lie on the straight line AA` which Coincides to axis PP` of Perspectivity .Since any Anti-point $A_E$ on Line PP` lies also on the circle of radius AA` , and since points P , P` lie on the same circle therefore points A` , P` , A_E coincide with PP` Axis of Perspectivity as in Fig1-(1).
B.. For Two points A,B perspective points A`,B`, lie on the straight line A`B` which is Parallel to axis PP` of Perspectivity . On Line PP` lie the Anti-points A_E , B_E which is the diameter *AOB of the circle ,* and whose points P , P` lie on the circle . The Infinite Axis PP` of Perspectivity are Coinciding to Perspective lines of points A`,B` and are also Symmetrical to the center O as in Fig1-(3).
C.. For Three points A,B,C not coinciding , perspective points A`,B`,C` lie on the straight line A`B`C` which is Parallel to axis PP` of Perspectivity . On Line PP` lie the Anti-points A_E B,E C,E , which line PP` is *Symmetrical to center O of the circumscribed to ABC triangle circle ,* and whose points P , P` lie on the circle . The Infinite Axis PP` of Perspectivity are Parallel to Perspective lines of points A`,B`,C` and also Symmetrical to center O as in Fig1-(3). *From above is seen that both Perspectivity and Projective - geometry are incorporated in Euclidean geometry and this because of the Anti-points of Material geometry .*

Because the New logic , of Material Geometry responds to Physical reality , the consistent Systems of the Non - Euclidean geometries - have to decide the direction of the existing mathematical logic. This is the why conversion factor , t = time, has not any essence in all universe , but it is a meter of changes only .

Since Time in Theory of Relativity is the main substance of Space - Time , then must be a quantity which has magnitude and direction and must follow the vector addition $\vec{a}+\vec{b}=\overrightarrow{ab}$. Unlike , the time intervals follow the Algebraic addition for scalar quantities a +b = t .

Proper time is measured between two events in GR Space-time and it is the Lorentz scalar , where there time , t , exists as a measure of changes in velocity and distance vectors of an isochronous Vectors-racing .

3.6. Waves and the exponential form of Monads

Angular velocity $\overline{w}$, and rotational momentum Λ in a cave conjugate and are represented as , $(\partial/\partial t , \overline{w})$ © (0 , Λ) = (- Λ , wxΛ) = $(-\overline{H}x\overline{\overline{P}},\square x\overline{\Lambda})$ = [λ ,□ x $\overline{\Lambda}$] . [13-15] . Since points A , B of [PNS] coincide with the infinite Points , of the infinite Spaces , Anti-Spaces and Sub-Spaces of [PNS] and exists rotational energy ± Λ and since Motion may occur at all Bounded Sub - Spaces (±Λ , λ) , then this Relative motion is happening between all points belonging to [PNS] and to those points belonging to the other Sub-Spaces (A ≡ B) . The Infinite points in [PNS] form infinite Units (monads) AiBi = $d\overline{s}$, which equilibrium by the Primary Anti-Space by an Inner Impulse (P) at edges A , B where $P_iA + P_iB \neq 0$, and it is , $ds = 0 \rightarrow N \rightarrow \infty$.

Monad , (Unit) $\overline{A}B$, Quaternion is the ENTITY and [A , B – $\overline{P}_A$, $\overline{P}_B$] ≡ Dipole =[⊕⊖]= □ = AB the LAW , the Material-point , so Entities are embodied with the Laws. Entity is quaternion $\overline{A}B$, and law |AB| = length = The Scalar and Real part which is the Space of points A,B and maybe any point A with its Pole B of rotation , and this since Space-curves are rolling on Anti-space curves , and Imaginary part the forces , $\overline{P}_A$, $\overline{P}_B$ or the Electromagnetic fields of AB . [58]

3.7. Cycloid → The Inner Isochronous motion of monads .

Isochronous motion of a point A , on cycloid happens in all Material-points , where the **y** axis reach **x-x** axis at the same time , regardless of the height from which they begin. This property is used for breakages on Common-circle ***Before*** these reach STPL line isochrones.

Isochronous motion on circles happens by the Rolling of equal circles[⊕⊖] in Material Point , due to Glue-Bond stresses {+σ-σ } with the same Period $T = \frac{2\pi}{w}$ dependent on angular velocity w , only , regardless of the position from which they begin rolling . This property is used for breakages on parallel ***After*** reaching STPL lines

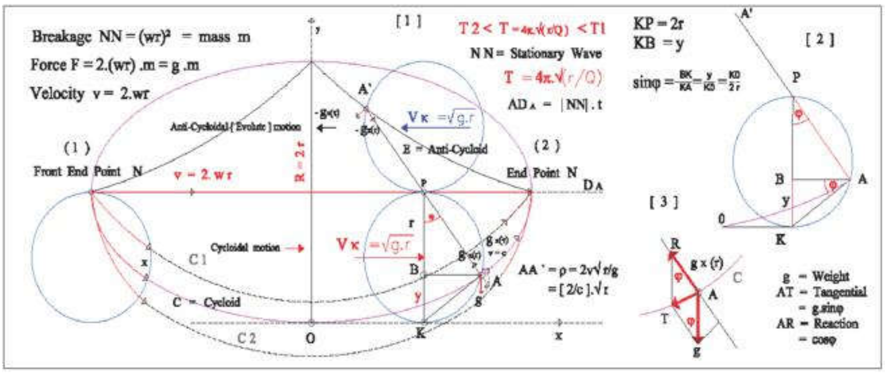


Figure.9.. *The Cycloidal motion in , Material Point ≡The monad is* Dipole ≡ [⊕⊖]=□=AA` *where* → A≡[⊕]

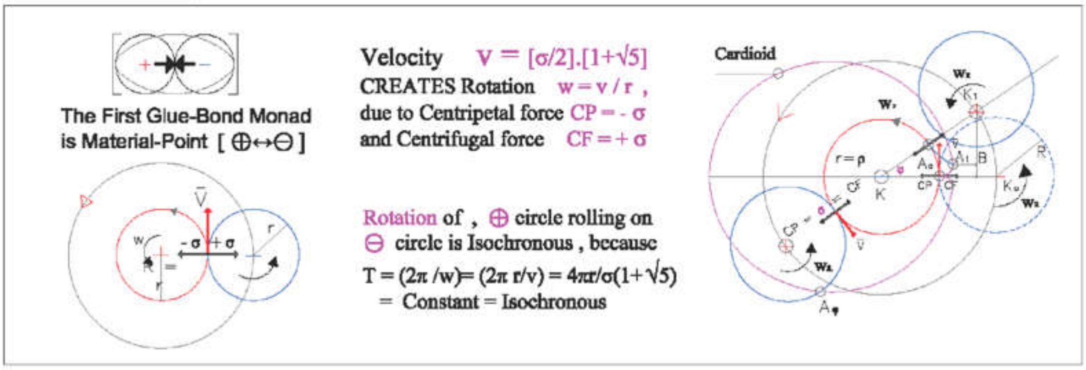
→A`≡[⊖] → |AA`|≡ □ ≡*The Brachistochrone Curve* C ≡ N1→N2. Motion on Curve C1 acquires a period T1 > 4π√r/g while on C2 T2 < 4π√r/g which is not Isochronous .

Motion on Curve C , *Cycloid* , acquires a CONSTANT period T = 4π√r/g which is Isochronous .

Monad (1) –(2) = NN The Electromagnetic Wave in NN , *is the energy distance* . Motion of point A on cycloid [C] , equilibrium from the opposite motion of point A` on Evolute {Anti-cycloid } . Vibration happens on AA` where the Mechanical motion (*the velocity* ,v) transformed to Electricity (*the Electromagnetic wave* E⊥P).

Space point A on cycloid [C] , is rolling on Anti-space point A` of Evolute curve as the Instaneous-Curvature Pole. [58] STPL line is the circular Rolling motion of , Space , Anti-space , is the cause of Vibration on the Instaneous Radius (diameter) of curvature centre of rotation through Sub-space , and or , on every couple of lines between Spaces and Anti-spaces .

Extrema case of , *Pascal`s line-rolling of any two circles* , is Euler-Savary mechanism where Instaneous -circle and Common-circle acquire the common Space , Anti-space Chord on where , *Rolling motion of the two curves is transformed to Vibration curves* .



igure.10. *The Isochronous Rolling of* circles[⊕⊖] (*in Material Point , due to Glue-Bond* {+σ-σ }) , happens 4πr

Properties (Fig.9) :

Cycloid is the curve described (traced) by a point P, on the circumference of a circle of radius , r, as this rolls along a straight line AA without slipping on an orthogonal coordinate system (x,y) at O . Let find the equation of this curve using the geometry logic in mechanics . Since total period of oscillation T= 4π√r/g and which does not depend on speed of rolling, (Huygens cycloid pendulum) but only from rolling radius ,r, means that the arc length l=8r is completed for faster , as one revolution in less time than the slower one , meaning that ,

On cycloid all points of y axis reach x-x axis at the same time , regardless of the height from which theybegin (isochrones) .This property is used for breakages to reach STPL line

In absolute magnitudes $\frac{dy}{dx} = \frac{KB}{KA} = \frac{BA}{BP} = \frac{BA}{2r-y}$ and $(BA)^2 = (BP).(BK) = (2r-y).y$ and

by squaring → $\left(\frac{dy}{dx}\right)^2 = \frac{y}{2r-y}$ …….(a) which is the differential equation of cycloid ,

and or as → $\left(\frac{dx}{dy}\right)^2 + 1 = \frac{2r}{y}$

For any element on trace ,ds , issues (a) and Pythagoras theorem as $(ds)^2=(dx)^2+(dy)^2 = \left(\frac{2r}{y} - 1\right).(dy)^2 + (dy)^2 = \left(\frac{2r}{y}\right).(dy)^2$ and $ds = \sqrt{2r}.y^{-1/2}.dy$ and by integrating ,

$$\int ds/dy = s = \sqrt{2r}.\int_0^y y^{-1/2} = \sqrt{2r}.\frac{y^{+1/2}}{-1/2} = 2.\sqrt{2ry} + C$$

and since in axis for y=0 *exists* s=0 and C=0, so **s**=2√KP. KB=2. √KA²= 2.KA = 4r.sin φ …(b)

i.e. the length of Cycloid curve , from point O to point A , is twice the segment of chord KA and when point A is at the end point (2) →

2.KA=4r for the semi-cycloid.

The area between the curve and the straight line is $A=3\pi r^2$ and the arc length l = 8r.

For motion on cycloid , we consider a Weight Q , at point A , moving with free motion . Since reaction N is vertically acting , doesn`t give any Tangential component therefore the only one becomes from Q which is equal to AT= g. sin φ, and since from (b) , $\sin\varphi = \frac{s}{4r}$ then $AT = g.\frac{s}{4r}$.

Since acceleration = $\frac{d^2s}{dt^2} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = -g.\frac{s}{4r}$ isochrones. Evolute also of a cycloid is a cycloid itself , (apart from coordinate shift) . Velocity vector of any motion is directed along the tangent and is the sum of the velocity vectors of the constituent motion , thus at each point A , of a cycloid , the line joining that point , to the point P, that circle is , then at the top of the generative circle is tangent to the Anti-cycloid and the line joining point A`, that is to that of bottom (of circle) is normal to the cycloid . Evolutes of a cycloid is the balancing cycloid, and called Anti-cycloid.

The *Tangential component of Acceleration* is $AT = g.\sin\varphi = \frac{g}{4r}.s$ and analogous to OA arc ,

While the *Centrifugal component of Acceleration* $\frac{v^2}{\rho}$, is dependent on initial point of motion. Any

Material point moving from A to P point , acquires velocity $v^2 = 2.g.PB = 2g(2r-y)$ and

$$\frac{v^2}{\rho} = \frac{2g(2r-y)}{2.PA} = g.\cos\varphi = g.\frac{PA}{2r} = \frac{g}{4r}.\rho \quad ....(e)$$

i.e. The *Centrifugal component of Acceleration is proportional to curvature radius* ,ρ, *with the same proportionality ratio* g/4r .

The velocity $v = \sqrt{g/4r}.\ \rho$ is proportional to curvature radius ρ, with proportionality ratio the root of g/4r.
on angular velocity $\bar{w} = \bar{v}/r = \bar{c}/r$ only and it is the Spin of particle |AA|.

Remarks **:**

then or $\{\ddot{x}=-w^2\dot{x}$ where w=} …(c) On cycloid , all moving points on y axis reach x-x axis at the same time (isochrones motion) Equation (c) is a Harmonic Oscillatory motion regardless of the height from which they begin showing that Acceleration is proportional to (they do not depend on the oscillation displacement and is directed towards the origin amplitudes) , or if , a particle of mass $m=|(wr)^2|$ with a period T= …(d) =1 tied to a fix point A executes a Simple harmonic motion under the action (Thrust) of since $w^2 =$ i.e. the tangential velocity $\bar{v}=\bar{w}.\bar{r}$, and since →linear Equation (d) denotes that the Harmonic momentum $\bar{p}$= [Breakage x Velocity] = $|\bar{w}.r|$. Oscillation due to any Force or Weight which $\sqrt{g/4r}.\ \rho = .\ \rho =$, then follows follows the free motion on cycloid , is a Cycloid`s trajectory with , a Total time period Independent of the amplitude of oscillation which is dependent and , is Isochronous **.**

a.. Breakage **x** Velocity = $\sqrt{gr.\rho.|\bar{w}|}$, and force $F = [(\bar{w}.r)^2.(\bar{w}.r)] = 2(mg/\bar{c}).\ \bar{w} = 2mg.(\frac{\bar{w}}{\bar{c}})$,
This property is used to show that the wavelength of norm $|\bar{v}|$, of vectors ,$\bar{v}$, is a Stationary wave, with the two edges as Energy material nodes , Cycloidally carried on wavelength |λ| = 2|A1-A2| twice the norm. In Fig.9 KA = 2.r. sin φ and KA.sin φ = y so $\sin^2 \varphi = y/2r$ and $\cos^2 \varphi = 1-y/2r = \frac{2r-y}{2r}$ and by division becomes $\frac{v}{\cos\varphi}=\sqrt{4gr}$,which means that any Weight falling , or rolling on Cycloid from upper point A , ratio $\frac{v}{\cos\varphi}$ remains constant , and for the center of PK $\frac{1}{2}\cdot\frac{v}{\cos\varphi} = \sqrt{gr}$, i.e. the rolling circle has a constant *velocity* and with an area of moving circle $A = \pi.r^2 = \pi.(2r.\cos\varphi)^2 = \pi R^2.\cos^2\varphi$.

b.. Thrust is the velocity vector $\bar{v}=\bar{w}.r$ on the circumference of common circle of the inversely rotating Space, anti-Space becoming from the points A,B,C of Space and A_E , B_E , C_E of Anti-Space . Because all velocity vectors AA,BB,CC carry material points A,B,C at points
D_A ,D_B , D_C ,in time ,t, isochrones , then material points follow a cycloid with period the norm of wavelength of velocities |AA|,|BB|,|CC| . Fig.5
This Simultaneity is succeeded by Lorentz factor where transformations between Inertial frames that preserve the velocity of light will not preserve simultaneously.

c.. Work W , by a constant force $F = 2(wr)^2$ exerted on an object [breakage $\pm(wr)^2$] which moves with a distance times $dx=|(wr)^2|$ is capable of Vibration and is calculated in two perpendicular Formulations (dx□dy) which is as, Stiffness k = N/m → velocity vector **v1** → *Electric field E* → and Flexibility f = m/N → velocity vector **v2** → *the Magnetic field* . For more in [39-40] . The why Energy is $v_K = v.\frac{r}{PA} = P$ transformed into velocity , and velocity to a field is explained also through Extrema Principle . [41]
Cycloid of Figure.9. is a cave and let this be IN Common-circle of STPL mechanism .

[1] *The applied force on this* NN *cave is*

E = h.f = **w**.(h/2π)= w . Spin.. and

rotational energy vector ±Λ of PNS. The **Spin** = .s²]/w = (r.s²)

wavelength of norm of velocity |v □| is the static equilibrium position vector of amplitude, ds, of dipole $|AB| =|\bar{v}|$ =ds and in terms of the static deflection, ds, then T =1/f= 2π/w where $ds = z = \bar{v} = A.e^{i.wt} = \bar{v}.\cos wt + i.\bar{v}.\sin wt$.
i.e. Breakages acquire different velocities and different energy , and because are following cycloid trajectories , thus , need the same time (isochrones) to reach [STPL] line. Simultaneity is a property of Absolute system and the intrinsic property of vectors and Poinsot's ellipsoid now becomes a → < Cycloidal ellipsoid >, since on c1(T1) > c > c2(T2).
Any material point [Medium-Field Material-Fragment] →[± s²]=$|\bar{w}x\bar{r}|^2$ →[MFMF]
Field following trajectory ,in=(c1), or ,out=(c2) , Cycloid=(c)=|A1-A2| needs more or less time T(2) < T = 4π √(r/g)<T(1) to reach end A2.
And since frequency f = 1/T and energy E = h.f then Cycloid motion Controls constancy of Energy by changing velocity, $\bar{v} = \bar{w}.r$, and period ,T, of monads.
Breakage quantity $2.(wr)^2$ under the tangential action $\bar{v}$ = wr becomes $2.(wr)^3$ acting on point A → 2wr.**m** of common circle. The same also for
$\frac{\mathbf{E}}{\mathbf{w}} = [\pm\bar{v}$

[2] **For E** = $\pm\bar{v}$ then →

Spin = $\frac{\mathbf{E}}{\mathbf{w}}$ = [± $\bar{\nabla}$.s²] / w =(± **r . s²**) →

Producing ± ***Fermions with spin*** $\frac{\mathbf{1}}{\mathbf{2}}$

[3] **For E** = [□i=2(wr)²=2.$\bar{v}$s²]=**2.(r.s²)** then

Spin =$\frac{\mathbf{E}}{\mathbf{w}}$=[2.$\bar{\nabla}$.s²]/w = **2.(r.s²)** → Producing

***Bosons of spin* 1**

i.e. Double energy [2.(r.s²)] on a constant cave creates **2** crests and doubling the frequency **(h) ,** with **Spin 1.**For **N-times energy [N.(r.s²)]** on a constant cave creates **N** crests N-times the frequency **(h)** with **Spin N/2** .

Since Energy in cave is an Electromagnetic Wave

[$\bar{E}$x$\bar{H}$] = Pressure = Spin S = ρ.c.w , or **[εE²+μH²] / 2 = 2rc. sin.2φ** → then Energy / sin2φ = **[εE²+μH²] / sin 2φ =** 2rc/ ρw = 4r² /ρ = constant , happening only on Cycloidal motion , where **ε**=Permittivity and **μ** =Permeability .

Above property happens in *Piezoelectric-effect* where *the Mechanical Energy* as { pressure or vibration} , executed on a material point or on a Dipole =[⊕⊖]=□=AB , *is converted into an Electric or transverse Magnetic wave* . [58]

Work from deformation is dW =$\frac{\sigma^2}{2E}$(dx.dy.dz).

It was shown that the Intensity is $I_d = \frac{\rho^2\pi^2c^3}{2\lambda^2}$, and for ρ=1 then is $I_d=\frac{\pi^2c^3}{2\lambda^2}$.[58]

Applying this to light-velocity-vector then Electromagnetic Wave {$I_d=\frac{\pi^2c^3}{2\lambda^2}$} in vector |c| , is creating a Mechanical deformation on Material point |c| { as ***Outer-Spin*** =$\frac{E}{w}$ = h.f }, which is then converted to an ***inner*** Electromagnetic Wave and which is recycled .

The linear electrical behavior of a Material point is , D = **ε** E , where D = the Electric displacement field , E = the Inside Electric field strength and then according to Maxwell`s equations □ . D = 0 , □xE = 0 and since in Elasticity , Hook`s law → s = m. σ m = Young modulus then ,

σ = 0 , s = □______u+u□ where u=displacement .2

All above when combined in *coupled equations* then s = m.σ +∂E and D = **ε** E +∂ σ .

In case of a Dipole =[⊕⊖]=□=AB in a Cave 2r , ON or OFF STPL , is [(+(wr)²)↔(-(wr)²)] and is oscillated in itself as ***monad .*** Fig.5-6 -12 , i.e. ***The Free vibration of monad*** AB =$\bar{q}$=[s+$\bar{v}$□i] oscillating under the action (a thrust) inherent in itself , subject to , *damping* , because energy is dissipated by the *stiffness* ,**k** , of monad and from a *constant of proportionality* ,**c**, regarding the motion of *mass* ,**m**, when placed into motion , the oscillation will take place at the natural frequency , f_n **,** which is a property of monad. ***For Displacement*** , x = AP_A ,***The homogenous differential equation of this motion is*** , mẍ+cẋ+kx = 0 ……. (1) which corresponds physically to the free damped vibration , where m = mass = a reaction coefficient to the change of velocity ẋ and k = stiffness = a reaction coefficient to the change of length ,|x|, x = the displacement , ẋ = velocity of monad , k and c constants as above ,with general solution given by the x(0) , ẋ(0) → A , B then displacement ,x, is , x = $e^{-i.(c/2m)t}$.[A.$e^{S.t}$ + B.$e^{-S.t}$] = = $e^{-i.(c/2m)t}$.[x(0).$e^{S\cdot t}$+ẋ(0).$e^{-S\cdot t}$] and

Oscillatory x = $e^{\pm i\sqrt{\left(\frac{k}{m}-\left[\frac{c}{2m}\right]^2\right)}\cdot t}$ =

$\cos\sqrt{\left[\frac{c}{2m}\right]^2-\left(\frac{k}{m}\right)} \pm i.\sin\sqrt{\left[\frac{c}{2m}\right]^2-\left(\frac{k}{m}\right)}$ .....(2) where , 1.. For coefficients $\left[\frac{c}{2m}\right]^2 > \left[\frac{k}{m}\right]$, no oscillations are possible , *is the Over-Damped ≡ The Particle like nature of monad .*

2.. For coefficients $\left[\frac{c}{2m}\right]^2 < \left[\frac{k}{m}\right]$ the exponent becomes an imaginary number and the terms are Oscillatory , *it is the Under-Damped ≡ The Wave like nature of monad ,* and this because of space rotation only ↻ .

3.. For $\left[\frac{c}{2m}\right]^2 = \left[\frac{k}{m}\right]$ then oscillatory , non-oscillatory and radical motion is zero , *It is the Critical Dumping in monads ≡ The Critical-Energy-Quantity →* CEQ *as in M-point . The Particle and or the Wave nature of monads ,or when →* $C_c = 2m\sqrt{\left[\frac{k}{m}\right]} = 2mw_n$ = 2.√ k.m a relation depending on the three reactions c , k , m .

***Electromagnetic fields of monads* :**

Any damping can then be expressed in terms of the critical damping by the non-dimensional number ζ = C/ and **S** in terms of ζ , $\left[\frac{C}{2m}\right] = \zeta\left[\frac{Cc}{2m}\right] = \zeta w_n$ is S = [-ζ± √(ζ²-1)].w_n and

the differential equation of motion becomes , ẍ + 2ζw_nẋ + w_n^2x = 0 ……(1- n) with the general solution given by the following three cases and equations ,

For $\zeta < 1$ is the Oscillatory motion , *The Under-damped case ≡ Wave like nature .*

$\bar{x} = e^{-\zeta.wn.t}.[A.e^{i\sqrt{(1-\zeta^2)}.wn.t} + B.e^{-i\sqrt{(1-\zeta^2)}.wn.t}] =$
$e^{-\zeta.wn.t}.\{\{[(\dot{x}(0)+\zeta.w_n.x(0)).\sin\sqrt{(1-\zeta^2)}.w_n.t] / [w_n.\sqrt{(1-\zeta^2)}]\} + x(0).\cos\sqrt{(1-\zeta^2)}.w_n.t\}\}$ ….(**2a**)

equation → $x = A.e^{s1.t} + B.e^{s2.t}$ where damped oscillation is equal to =
$s1,2 = -[c/2m] \pm \sqrt{\left[\frac{c}{2m}\right]^2 - \left(\frac{k}{m}\right)}$ and and according to Planck`s formula E $w_n.\sqrt{(1-\zeta^2)}$ w $= h.f_n = h\{\frac{n}{2\pi}\}$

$S = , \sqrt{\left(\frac{k}{m}\right) - \left[\frac{c}{2m}\right]^2}$ represents the energy in monads .

a constant coefficient , **For** $\zeta > 1$ is the Non-oscillatory motion , the and for initial conditions *Over-damped case ≡ The Particle like nature*

which indicates that the frequency of the $w_d = \frac{2\pi}{\tau d}$ with the two roots increasing and decreasing with the general solution , $\dot{x} = \vec{v}\, x = A.e^{[-\zeta+\sqrt{\zeta^2-1}].wn.t} + B.e^{[-\zeta-\sqrt{\zeta^2-1}].wn.t}$ where
A $= \{\dot{x}(0) + [\zeta+\sqrt{(\zeta^2-1)}].w_n.x(0)\} / [2w_n.\sqrt{(\zeta^2-1)}]$
B $= \{-\dot{x}(0)-[\zeta-\sqrt{(\zeta^2-1)}].w_n.x(0)\} / [2w_n.\sqrt{(\zeta^2-1)}]$

(**2b**) which indicates that the frequency of the damped oscillation is equal to $w_d = \frac{2\pi}{\tau d} =$
$w_n.\sqrt{(1-\zeta^2)}$ and according to Planck`s formula $E = h.f_n = h\{\frac{w_n}{2\pi}\} = h\{\frac{\vec{v}}{2\pi r}\} = M.v_n$, *and since also* $\vec{v}_n = w_n.r$, *and* M = The complex mass , thus represents the Kinetic energy in monads depending on velocity $\vec{v}_n$ and M.

For $\zeta = 1$, is the Internally Isochronal oscillatory motion , (*the Inner cycloidal motion of monads*) is *The Extrema , critical damped motion case* and displacement ,**x**, is as →
$x = e^{-wn.t}.[A + B.t] =$
$= e^{-wn.t}.\{x(0) + [\dot{x}(0) + x(0).w_n].t\}$ ……(**3b**) i.e. a double root $S1 = S2 = -w_n$ which is according to the Newton`s second law , the deformation of the real part ,|s|, which is $k.|s| = -w = -mg$ and frequency $f_n = (1/2\pi).\sqrt{g/|s|} = 2\pi\sqrt{m/k}$ depending on the mass and stiffness of monad , being its properties.
Above indicate that Extrema-frequency of this critical damped oscillation is equal to $w_d = \frac{2\pi}{\tau d} = w_n.\sqrt{(1-\zeta^2)} = 2\pi.f_n$ and according to
Planck`s formula $E = h.f_n = h\{\frac{w_n}{2\pi}\} = h\{\frac{\vec{v}}{2\pi r}\} =$
$M.v_n$, *and since also* $\vec{v}_n = w_n.r$, *and* M = The complex mass , then represents the Kinetic energy in monads depending on velocity $\dot{x}(t) = \vec{v}_n$ and M , and for any position , x = x(t) , of vibration , and → When Velocity $\dot{x}(t)$ is , $\dot{x}(t) > 0$ then the type of response is Over x , *is a scalar quantity*} and Energy-System quantity is stored in velocity vector $\vec{v}_n$ by virtue of its ***velocity- vector-cave*** and not in the scalar quantity .
b.. In the absence of velocity vector , mass is not existing {*mass ,which is the reaction to the constant velocity change , is zero* } and Energy-System is stored in velocity vector $\vec{v}_n$ by virtue of its ***velocity -vector - cave*** , although the scalar quantity is zero ,
And for the Energy partly ***Potential U ,***
In the Absence of velocity vector , mass is not existing {*mass ,which is the reaction to the constant velocity change , is zero* } and Energy is stored in velocity vector $\vec{v}_n$,
a.. ***in the form of Strain-energy in Elastic Deformation for the Work done and which is a Force-field for Solids***] which is reverted to an Electromagnetic field .
b.. ***Strain-energy in monads is the Velocity-Cross-Product-vectors in the Homogeneous Deformation of the Work done and which is an Electromagnetic-field in the*** $|\vec{v}_n|$, ***Stationary - Wave – cave***] .
In [22-23] , any Monad NN = N(1) ↔ N(2) is the dipole , ($P_1 \leftrightarrow P_2$) , or **[**{N(P_1) ← 0 → (P_2)N}**]**
It is the symbolism of the two opposite forces (P_1) , (P_2) which vibrate perpendicularly in monad (*Resonance-cave with an Electromagnetic Response*) and are created Mechanical forces at the edge points N1 , N2 .
Balancing of Monads ≡ *Quaternions* , happens on Evolutes *Cycloid* , *Anti-cycloid* . For velocity $\bar{v} = \bar{c}$ = light velocity , curvature radius is $\rho = XX` = 2c\sqrt{r/g}$ , and Spin S is , $\bar{S} = \bar{\nabla}x(\tau) \times XX` = [g.\sin\varphi].\rho = [g.\sin\varphi].2c\sqrt{r/g}] = 2c.\sin\varphi.\sqrt{rg}$
$= 2c.\sqrt{r.g}.\sin\varphi$, ***i.e.***

Energy → **c ,** *as Spin* $\bar{S}$***, is Unified with the***
$\dot{x}(t) = 0$ then the type of response is From x , ***Space-Energy*** *as radius ,r.*

$\vec{v}_n$ it follows existence of mass , m , also { *m =* rotation of the positive ,+, to the negative ,-, *mass = the reaction to the velocity change , which* with Lever-arm Displacement , AP_A on AA_o.

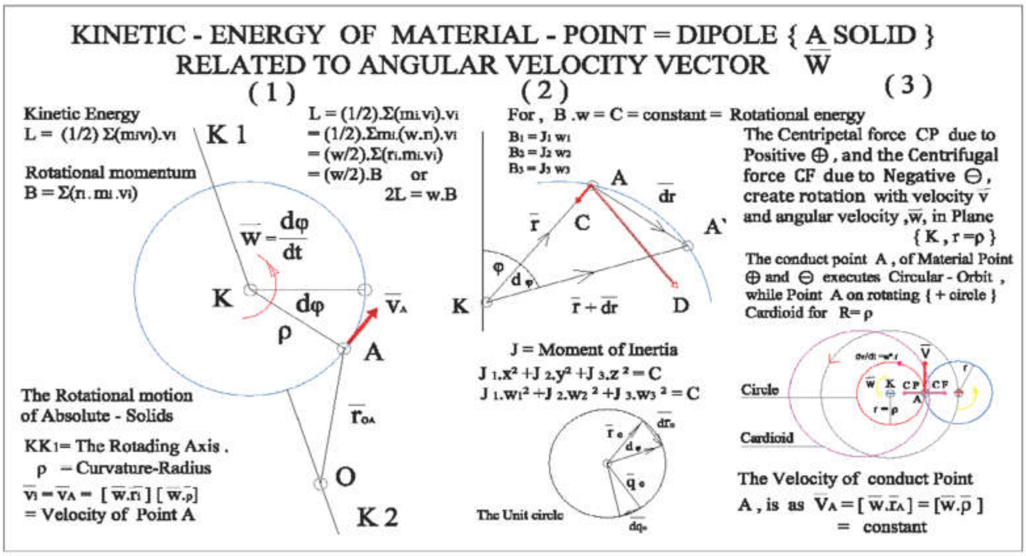


Figure.11. *The Cycloidal motion in , Material Point ≡The monad is* Dipole ≡ [⊕⊖] =□= $K_r A K_{R=r}$ *where →* K_R≡[⊕]↔ K_r≡ [⊖] → ≡ 0 . The total torque becomes from ± Spin which equilibrium in System of *circle* K_r. *Evolute circle* K_R , as *Cardioid of the same center.*

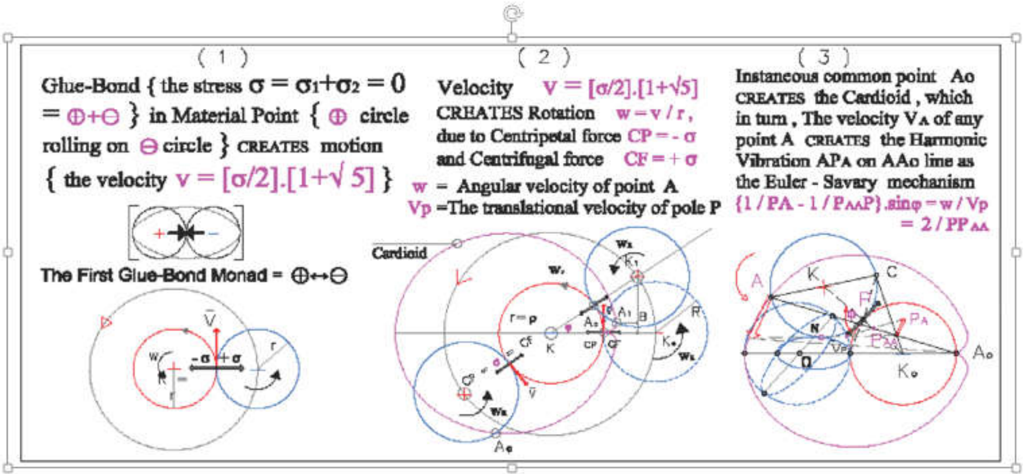


Figure.12. Pole of rotatation P , **on STPL line A** P_A**,** is the Instaneous centre of rotation for [⊕] , Space on [⊖] Anti-space , through Sub-space ,□, and or , every couple of lines between Spaces and Anti-spaces . Cardioid is the envelope of circles (K_o,R) , (K ,r=R) whose centres K_o , K lie on a given circle (P,PAo) which pass through a fixed point , Ao , on the given circle (P , PAo) .Nutation happens at ,N, common point. Analytically in [58].

In (1) GLUE-Bond becoming from opposite ± stresses σ1= - σ2 and create Velocity $\bar{v} = \frac{\sigma}{2}\ [1+\sqrt{5}]$
In (2) velocity $\bar{v}$ = w.r creates Rotation which becomes , according to Newton`s third law, from the Centripetal ,CP, and the Centrifugal force ,CF, and w is the angular velocity of point A. In (3) velocity v_A=w.(APAA) of point A creates the Free Harmonic Vibration on AP monad following the Euler-Savary mechanism where , *Rolling motion is transformed to known Vibration curves .*

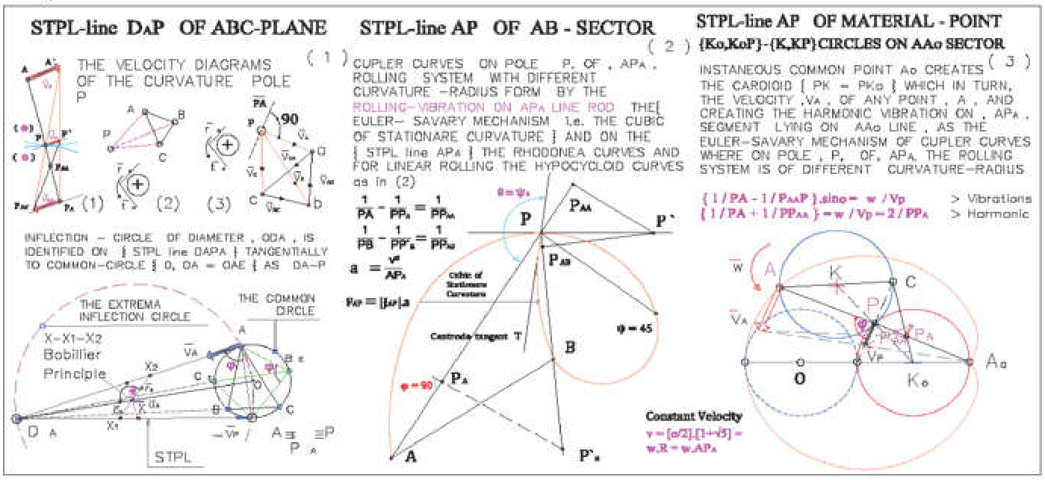


Figure.13. The STPL line , In *a Material point* {⊖≡ K_o,K_oP – ⊕≡K , KP} , In *a Material-Segment* AP} , *and* In *a Material-Plane triangle* { ABC } is as in (3),(2),(1) .

(±) Breakages { *in STPL lines* } become the *, Vibrating Curves of Material points* .

In (1) STPL line ***of Plane ABC*** , *extrema* D_AP_A≡D_AP≡D_AA_E ≡ ,r, is tangential to Common-circle , and ***Inflection- circle*** passes through Space-point , A≡[⊕] , Anti-space point A_E≡ [⊖] ,which coincides with the Instaneous curvature-centre of rotation , *the Pole P* , and thus forming the material angle ,) . t , on angle < AD_AP .

$$\vartheta = \vartheta_A . t = (\frac{v_A}{\sqrt{c^2-r^2}}$$

All chords through the Sub-space Plane-triangle A D_AP , follow Bobillier-Principle for curvature centres D_A and CREATE the Vibrating Energy-Geometry-Segments D_AA ,D_AP .

Velocity of point A , is v_A=w_A. r_A , *where* , w_A= the angular velocity of point A , r_A= the distance , AP, between the moving (+) point A and (-) point P the Pole .

In (2) STPL line , ***of sector AB*** *which two points , A , B* , are OFF Common-circle , and lie on the circumference of Envelope-circles , *O,OA=OB* , with the common Anti-space point , P(-) , and thus forming the material angle , $\vartheta = \vartheta_A$. t = ($\frac{v_A}{\sqrt{c^2-r^2}}$) . t , with centrode tangent T.

Euler-Savary mechanism establishes the relation among points A , P, P_A and P_{AA} and CREATE the Envelope curves , *Stationary-curvature paths* , generated by the Vibrating →Velocity-Energy - Geometry Segment A P_A , on AP line .

In (3) STPL line , **of *Material point AP*** , {⊖≡ K_o,K_oP – ⊕≡K , KP} of Space point A (+) and Anti-point P(-) is rotating through point A_o , which is the center of common-circle and forming the material angle , $\vartheta = \vartheta_A . t = (\frac{v_A}{\sqrt{c^2-r^2}}$) . t , CREATE the Cardioid-Envelope curves generated by the above Vibrating Velocity-Energy-Geometry-Segment A P_A , on AA_o rotating line .

Remarks in Figure 13 :

a.. It was shown in [58] that *Clashed Breakages which are located* IN *the STPL Cylinder , Acquire Oscillation from their inherent Vibration as in* (1)
–(2) *, and consist the Moving Particles while , Un-clashed Breakages located* OUT *the STPL Cylinder , Acquire Oscillation from their between Glue-bonding and consist the Rest Particles* (3).

In all cases , STPL line mechanism consists the
< Energy-Geometry-Length ≡ Quantum ≡ AP > The Space as Velocity-vector-energy V , in the cavity of the Common-circle of radius ,r, and constant angular velocity ,w , is transported as Energy from point A to Pole P , coinciding , with Point P as P ≡ A_E≡ P_A , where then the two conjugate points ,T,J, lie on STPL line as Pascal`s P_A and Desargues D_A points with the constant angle < φ = < D_AAP_A ≡ < D_AOA , on Common circle and on Extrema circle .

b.. Since all properties of Physical entities exist only in Pairs and exists the scientific notion that < Opposites Attract > < Similar Repel > then ***considering*** , Material point A≡[⊕]Anti-space point A_E≡[⊖] or AA_E = AP as a ***Physical System*** which has only one physical property which is ***Stress ≡ σ*** can predict measurements produced , and also results which are according to the Newton`s second law , the Forces of Circular motion and tangential and angular velocities $\bar{v}$,$\bar{w}$, **$\bar{v}$= w.r** which is the ***Hidden-variable of the System*** . This continuously equal velocity $\bar{v}$, creates on any Material-point [Point , A–Anti-point P=A_E] ≡{Energy-Geometry-Length ≡ Quantum ≡ AP } the envelopes of Cardioids which are of Wave function , whose domain is the configuration space in ***Material-point -energy-equilibrium***. Since also an

Isolated system does not loses or gain Energy so , this Material-point is self consisted and constitutes , ***The First Eternal < Self – Moving – Energy – Dipole > ≡ The Quantum , of this cosmos .***

c.. It was proved in [58] that , in case of a curve rolling on its constant envelope curve , then the curvature center of the envelope curve coincides to that of the rolling curve .

In Figure 12-3,Euler-Savary mechanism on AP is

$$\left[\frac{1}{PA} - \frac{1}{Paa\,P} \right].\sin\varphi = \frac{w}{Vp} = \frac{Angular\ Velocity}{Tangential\ Velocity}$$

, i.e. a *Geometry-energy-motion relation in the Material-Point* , where energies become from , **w** , is the angular velocity of point A and $\mathbf{v_P}$ is the translational velocity of pole P , and Creating the curves , Free Harmonic Vibration .

d.. It was shown in [14-16] that , *in The Elastic material Configuration the Strain energy is absorbed as Support Reactions and displacement field in the three dimensions* [□**ε** (u̅,v̅,w̅)] *upon the deformed placement* , (these alterations of shape by pressure or stress is the equilibrium state of the Configuration) , *as* G.□². **ε** + [m.G / (m-2)] **.** □ [□. **ε**] = F , *where*

E = Young modulus of elasticity.

G = Shear modulus = E.m/2(m+1) m = Poisson`s ratio = 1 / μ ≈ 10/3 σ = Stress = Force / Area.

ε = Strain = change of length / length .

F = External forces .

The linear electrical behavior of a Material point is , Ď = **ε** Ê , where Ď = the Electric displacement field , Ê = the Inside Electric field strength and then according to Maxwell`s equations □. Ď = 0 , □xÊ = 0 and since in Elasticity , Hook`s law → **ε** = E . σ and then ,

. σ = 0 , **ε** =□______u+u□ *where* u=displacement .

2

All above when combined in *coupled equations then →* **ε** = E . σ +∂Ê and Ď = **ε** Ê +∂ σ …(1) and since in Material-point σ =2(1+√5).v̅ = constant , *since v = w.r* , then (1) becomes , **ε** = E . σ +∂ Ê = 2.E(1+√5). v̅+∂Ê Ď = **ε** Ê +∂σ = **ε** Ê +0 = **ε** Ê ………(2)

System (2) defines the Strain **ε** ,and the Electric displacement field Ê = [⊖] ,in Material-point .

.. The Geometry of STPL .

In Figure.3-(3), the tangents at points A,B,C formulate triangle $K_A\ K_B\ K_C$, the inscribed to it largest circle O,OA=OB=OC , which incenter is the intersection of the three internal angle bisectors at K . Because the internal bisectors of angles are perpendicular to their external bisectors , it follows that the centers of the incircle together with the three excircle centers form an orthocentric system .On this coordinate system is possible any geometrical analysis. By using the Trilinear coordinate system on ABC Space –triangle then for , Incenter is → 1 : 1 : 1

Incentral triangle Vertex opposite B = 1 : 0 :1

Incentral triangle Vertex opposite C = 1 : 1 : 0

External triangle Vertex opposite A = -1 : 1 : 1

External triangle Vertex opposite A = 1 : -1 : 1

External triangle Vertex opposite A = 1 : 1 : -1 Defining the lengths

a = KB KC , b = KC KA , c= KA KB , The STPL mechanism is the Mould consisted from any Common circle O,OA=[OA`≡ OAE], O,OB = [OB`≡ OBE], O,OC = [OC`≡ OCE], and the common lines DA-PA , DB-PB , DC-PC all on a line of STPL . On the infinite sectors ADA-APA , BDB-BPB , CDC-CPC vibrate the breakages [± s² = ± (wr)²] and [□i = 2(wr)²] forming all

Excenters is → -1 : 1 : 1 , 1 : -1 : 1 , 1 : 1 : -1 Coordinates for point O are,

$$\frac{b+c-a}{a} : \frac{c+a-b}{b} : \frac{a+b-c}{c}$$

Incentral triangle Vertex opposite A = 0 : 1 : 1

$\frac{a+b+c}{2}$ ***families of curves and the Euler-Savary*** d=[ ]=The semi-perimeter then ***Coupler-curves of the Cubic-Of-Stationary - Curvature mechanism of Space , Anti-space*** Inscribe radius r = $\frac{\sqrt{d(d-a)(d-b)(d-c)}}{d}$ = |OA| ***Vibrating end-curves .*** Coordinates for point K are, Dimensioning of the mechanism is possible

$$\frac{bc}{b+c-a} : \frac{ca}{c+a-b} : \frac{ab}{a+b-c}$$

by using analytical geometry.

Synopsis 2 :

Point in E-Geometry , which is nothing and dimensionless , i.e. the Zero , can be derived from the addition of a Positive (+) *and a Negative* (-) *number , while* ***Material point has dimension*** **,ds,** ***and Energy the Work W*** *, the Segment* **ds** = [⊕⊖] *and Work* **W** = P.ds , *and originates in the the same way . Adding it to numbers i.e. to Monads ,* ***creates Primary Particles , the Rest-Gravity constituent and the Atoms of the Periodic System*** *in Planck`s Space-Level .Monads are Spinning because of the* ***Inner Electromagnetic Waves*** , E ⊥ P *, which* ***create External Spin*** *and again the* ***Inner Electromagnetic Waves*** , E - P , *continuing this eternal Cycle .*

In Mendeleyev`s Periodic Table , chemical properties of the elements are a periodic function of their atomic weight and in [58] was shown that ***, any Next-Atom Energy , is equal to Prior + the distributed.*** *Since* all material points are produced from (±) Breakages which consist the
[⊕ ,⊖ ,☼] Energy-Units as follows ,
Breakage $s^2 = +(wr)^2$ = The Positive ⊕ Unit ,
Breakage $-s^2 = -(wr)^2$ = The Negative ⊖ Unit ,
[⊕↔⊖] = □ =The Rest Energy Quanta ≡ 0 the Zero Unit ,
Breakage $2s^2 = 2(wr)^2$ = The Energy ☼ Unit , then

Primary Segment of Material-point *is of the* ***Form*** [⊕↔⊖] = □ = 0 *, and its* ***Content*** $\bar{v} = \frac{\sigma}{2}[1+\sqrt{5}]$
Finite |⊕↔⊖| *and* ***Infinite*** , $\bar{v}=\bar{w}.r=\infty.0$ *, to all Monads* $L_v = e^{i.\left(\frac{N\pi}{2}\right)}$ b=10⁻N= − ∞ *, and for* N≡$\bar{w}$=∞ → 0 *, the* ***Atraction*** [⊕↔⊖]*and the* ***Repulsion*** ⊕→ ←⊕ *, the* ***Quantity*** *in Real part* Form AB=L_v=|⊕↔⊖| *and in Imaginary part* [⊕↔⊖]=0 *, and the* ***Quality*** [⊕↔⊖=σ] ≠ 0 *by differentiation , and so on .*
Since also ***Imaginary Part*** *is always* [⊕↔⊖] = 0 *then Form and Content are absolutely inseparable and pass from zero for all Opposites , so all Entities are embodied with the Laws , and since also valid* [⊕↔⊖] ≠ 0 *then , the* ***Zero equality is the Constant and Critical-Energy-Quantity*** → CEQ *and is* {{ Stress , σ = CEQ is Producing velocity **$\bar{v}$= w.r** , and consists the ***Hidden-variable of this tiny and Self– Moving– Energy– Dipole , System*** }} *, for any transition in Quality , a kind of Constant-Catalyst which is not changing the composition of Primary Material-Segment ,* the unity of opposites *and also the Work ≡ Energy involved in all levels . In this way in nature nothing remains constant because by changing ,w,r, in an eternally existing constant velocity vector* **$\bar{v}$** *then everything is in a perpetual state of transformation , motion and change . The Rest Energy-Quanta acquire a Resistance in motion which is Stress ,* σ = CEQ *, i.e. a meter , a number measuring this magnitude and it is that what is called* ***Matter*** *which has nothing to do with energy.* **GR *considering Energy and Mass equivalent*** *creates a great confusion because , Energy is motion it is* ***Content*** $\bar{v} = \frac{\sigma}{2}[1+\sqrt{5}]$≡[⊕↔⊖] *, while Mass* ***is a Number*** *measuring the changes in velocity-vector motion* |**$\bar{v}$**| *, and it is the law ,* ***while Content*** |AB| ≡ |$\bar{v}$| ≡ [⊕↔⊖]≡ *The Energy length (the energy-quanta) of opposite points* |A ,B| .
In Primary-material-point , ***Form*** (r) ***and Content,*** [⊕↔⊖] *, is constant while in all others issues the law of transformation of* ***Quantity into Quality*** *, extended from the smaller particle to the largest phenomena . Since Material-point is of Form* [⊕↔⊖] = □ = 0 it is with binding points with no energy released . Since mass is the meter of Energy-velocity-vector changes , then this meter cannot exceed the frequency of light-velocity . The why light-velocity $\bar{v}$ *is the maximum and constant in* [58] . ***Changing the Form***(r) *means much more the Content ⊕ or ⊖ , or* [⊕↔⊖] ≠ 0 *,* ***is Negative-Energy ,*** *while the ,* ***Changing of Content ,*** *is an increasing in frequency which occurs in standing-waves and where then decreases the reaction to the motion (the mass) , because* $v = w.r = \frac{2\pi r}{T} = 2\pi r.f$ = constant .
It was shown in [58] *that ,* ***any Next-Atom , Energy*** *, is equal to Prior + the distributed i.e.* ***the law of Quality and Quantity*** *.The same also in Chemistry from gas to liquid or solid which is usually related to variations of temperature and pressure .*
Anti-Energy or Negative-energy is not existing because it is the Difference between the two (+) ≡ ⊕ *and* (-) ≡ ⊖ *Contents , in Energy-Form , i.e. it is a meter of the difference between the two magnitudes .*
Energy *, motion , and* ***the reaction to the change of velocity-vector*** *, mass , are absolutely inseparable.*

B.. *How , The Energy from Chaos becomes Monad .*
1.. General :
It was shown [33-36] that Un-clashed Fragments through center O , consist the Medium-Field Material-Fragment →[± s^2] = [MFMF] as base for all motions , and Gravity as force [□i], while the clashed with the constant velocity ,$\bar{c}$, consist the Dark matter [± $\bar{c}$.s] and the Dark energy [$\bar{c}$.□i] , or from → Breakages[± $s^2 = \pm (wr)^2$] and [□i = $2(wr)^2$] where then become Waves { *Distance* ds = A A_E *is the Work embedded in monads and it is what is vibrated* } with Vibrating equations of motion becoming ,
A → Particles, with Inherent Vibration ,
B → Gravity-field-energy, without Vibration
C → Dark-matter-energy constituents and as ,
A.. [±$\bar{v}.s^2$] → Fermions and [$\bar{v}$.□i] → Bosons ,
B.. [± s^2] → [MFMF] Field, and the binder , Field is [□i] → Gravity force ,
C.. [±$\bar{c}.s^2$] → Dark matter , and the binder Gravity force [□i] , [$\bar{c}$.□i] → The Expanding Dark energy.
From above is seen that in ,
A *, and , C , case { Energy as velocity ,* $\bar{v}$ *,} exists in the Discrete monads ,* ± $\bar{v}.s^2$ *and* ± $\bar{c}.s^2$.
B ***case , is the transportation of Energy , from Chaos to Material points*** [$+s^2 \leftrightarrow -s^2$]. The How ??
C
2.. The Kinetic - Energy in Material- Point and the Central Axial-Ellipsoid .

It was referred that the Constant and Critical-Energy-Quantity becomes from Stress , σ , between the two opposite Contents [⊕↔⊖] which in turn Produces velocity $\bar{v} = \frac{\sigma}{2}[1+\sqrt{5}]$ and $\bar{v}$ in turn the angular velocity $\bar{w}$ on ⊖ sphere of radius ,r, where $\bar{v}$= w.r , and consisting the Hidden-variable of this tiny *Content , which is a* **Self –Moving –Energy–System** . The circular Rotational-motion of this
⊕ Material-sphere on the ⊖ sphere , is as that of a Rigid-body . [57]
Following Euler-Lagrange classical-mechanics for the solution of equations in tautochrone-problem and Energy expressed as a function of positions and velocities , i.e. *Space-Energy-motion* then exist ,

2.1. ***Angular Velocity and Rotational Kinetic Energy .***
In Figure.12-3 and Figure.13 , ⊕ sphere is composed of (i) material-points A_i , of discrete mass m_i=1 rotating with velocity ,$\bar{v}_i$, about center-point ,O, of ⊖ sphere and angular velocity w_i and ,w, the angular velocity for the center of mass K . Mass m_i is constant ,m, at every point of ⊕ sphere.
In Mechanics , Kinetic-Torque is identity $\frac{d}{dt}[r\bar{v}]$ {=$[\frac{dr}{dt}\bar{v}]$ +$[\bar{r}\frac{d\bar{v}}{dt}]$ = $[\bar{v}\,\bar{v}]$ + $[\bar{r}\frac{d\bar{v}}{dt}]$} = $[\bar{r}\frac{d\bar{v}}{dt}]$ and since $\bar{v} = \frac{\sigma}{2}[1+\sqrt{5}]$ and $\frac{d\bar{v}}{dt} = \frac{d\sigma}{2}[1+\sqrt{5}]$ then $\frac{d}{dt}[r\bar{v}] = \frac{d}{dt}[r\frac{\sigma[1+\sqrt{5}]}{2}] = \frac{d}{dt}[\frac{r.\sigma[1+\sqrt{5}]}{2}] = [r.\overline{dw}]$=[r.Force]=[r.F]
Momentum $\bar{B}$= [r.mv] and Moment $\bar{M}$ = [$\bar{r}$. $\bar{F}$] , so Moment → $\bar{M} = [\bar{r}.\bar{F}] = \frac{d\bar{B}}{dt}$, i.e.
The Moment $\bar{M}$ of the moving force $\bar{F}$, from a constant point ,O, is equal to the change of Momentum to , O center , and for (i) points ,

Rotational momentum is expressed as → $\bar{B} = \Sigma[\bar{r}_i.\, m_i\bar{v}_i]$
Rotational Velocity , is expressed as → $\bar{v}_i = [\bar{w}.\bar{r}_i]$
From above equations is defined a) the momentum of , ⊕ sphere as the Summation of linear momentum $m_i\bar{v}_i$ of material points A_i rotating about center ,O, and b) the velocity $\bar{v}_i$ for every point A_i related to angular velocity $\bar{w}$ of mass-center K , as ,
$\bar{B} = \Sigma m_i.\{\bar{r}_i.[\bar{w}.\bar{r}_i]\} = (\Sigma m_i.r_i^2)\bar{w} - \Sigma(m_i\,\bar{r}_i\,.\bar{w}\,\bar{r}_i)$ ….. (1)
The referred magnitudes m_i , $\bar{r}_i$ of the ⊕ sphere and center-point ,O, are fixed to ⊖ sphere. Considering {$\bar{B}$ and $\bar{w}$} as ***two rotating vectors*** {$\bar{\rho}$, $\bar{a}$ } then , (1) becomes $\bar{\rho} = (\Sigma m_i.r_i^2)\,\bar{a} - \Sigma(m_i\,\bar{r}_i\,.\,\bar{a}\,\bar{r}_i)$ ….. (1a) , then , $\bar{\rho}^{\bar{a}} = (\Sigma m_i\, r_i^2)\, a^2 - \Sigma\, m_i(\overline{ar_i})^2$ and since $\overline{ar_i} = a.(\bar{a}_o\,\bar{r}_i)$ where $\bar{a}_o$ is the unit vector on radius $\bar{a}$ then , $\bar{\rho}^{\bar{a}} = a^2\, \Sigma m_i\, [\, r_i^2 - (\bar{a}_o\,\bar{r}_i)^2\,]$
The meaning of terms $\bar{a}_o$, $\bar{r}_i$, $\sqrt{r_i^2 - (\bar{a}_o\,\bar{r}_i)^2}$ are shown and the last one denotes the distance of point A_i from $\bar{a}$ axis , therefore $\bar{\rho}^{\bar{a}} = J_a.a^2$, where J_a is Moment of Inertia of ⊕ sphere and ,a, distance related to , $\bar{a}$, axis . Denoting as moment of inertia of ⊕ sphere to (OKoK) Plane, the Sum of products of $m_i\bar{r}_i^2$, $\bar{r}_i$ perpendicular to , (OKoK) plane then , $\Sigma m_i.(\bar{a}_o\,\bar{r}_i)^2 = J_{(a)}$
Considering rotating vectors {$\bar{\rho}$,$\bar{a}$} as in (1a) the changeable vectors {$\bar{B}$,$\bar{w}$} become $\bar{B}\bar{w} = J_w w^2$ ..(2a) where J_w= the moment of inertia to Instaneous axis of rotation .

Since also the *Rotational Kinetic Energy* $L = \frac{1}{2}J_a w^2$, then → $\bar{B}\bar{w}$ = 2 L
In a three Dimensional Coordinate –System where $r_i^2 = x_i^2+y_i^2+z_i^2$
$(\Sigma m_i.r_i^2) = \Sigma m_i.(\,x_i^2+y_i^2+z_i^2\,)$ and $\Sigma m_i\,(y_i^2+z_i^2\,) \equiv J_x$, $\Sigma m_i\,(z_i^2+x_i^2\,) \equiv J_y$, $\Sigma m_i\,(x_i^2+y_i^2\,) \equiv J_z$ …(3) where J_x , J_y , J_z are the moments of Inertia of, ⊕ sphere , on the three ,x,y,z axis and (3) to the three Planes , becomes → $\Sigma m_i x_i^2 = J_{(x)}$, $\Sigma m_i y_i^2 = J_{(y)}$, $\Sigma m_i z_i^2 = J_{(z)}$ ……………(3a) where
$J_{(x)}$, $J_{(y)}$, $J_{(z)}$ are the moments of Inertia of, ⊕ sphere , to the three Planes , i.e. to the yz , zx , xy .
$\Sigma\, m_i.\, y_i z_i = J_{yz}$, $\Sigma\, m_i.\, z_i x_i = J_{zx}$, $\Sigma\, m_i.\, x_i y_i = J_{xy}$ ……………(3b)
Magnitudes J_{yz} , J_{zx} , J_{xy} and the equivalent J_{yz} , J_{xz} , J_{yx} are the *Diverted-Moments* or Centrifugal to Planes (y)-(z) , (z)-(x) , (x)-(y) respectively .
$\Sigma\, m_i r_i^2 = \Sigma m_i\,(x_i^2+y_i^2+z_i^2) = J_p$ ..…………(3c)
where J_p magnitude is the Polar-moment of Inertia to center , O . Equalities are proved as ,
$J_{(y)}+J_{(z)} = J_x$, $J_{(z)}+J_{(x)} = J_y$, $J_{(x)}+J_{(y)} = J_z$ and $J_x + J_y + J_z = 2\, J_p$ ……(3c) where x, y, z are projective of vector , $\bar{a}$, on coordinate axis and x_ρ , y_ρ , z_ρ those of vector $\bar{\rho}$.
Projective vectors of (1a) on , x , axis is holding $x_\rho = (\Sigma m_i\, r_i^2)\, x - \Sigma\, m_i\, x_i.\overline{ar_i}$ and since $\overline{ar_i} = xx_i + yy_i + zz_i$, then → $x_\rho = x\,\Sigma m_i\,(r_i^2 - x_i^2\,) - y\,\Sigma m_i\, x_i\, y_i - z\,\Sigma m_i\, x_i\, z_i$ and according to symbolism then → $x_\rho = x.J_x - y.\,J_{xy} - z.\,J_{xz}$. Analogically to ,y, and ,z, axis exists,
$x_\rho = x.J_x - y.\,J_{xy} - z.\,J_{xz}$, , $y_\rho = -\,x.J_{yx} + y.\,J_y - z.\,J_{yz}$, $z_\rho = x.J_{zx} - y.\,J_{zy} + z.\,J_z$ ……..(4)
Working Analogous on $\bar{B}$, $\bar{w}$ vectors then ,
$B_1 = J_x w_1 - J_{xy} w_2 - J_{zx} w_3$, $B_2 = J_{xy} w_1 + J_y w_2 - J_{yz} w_3$, $B_3 = -J_{zx} w_1 - J_{yz} w_2 + J_z w_3$ …..(4a) where B_1 , B_2 , B_3 w_1 , w_2 , w_3 Projections of vectors $\bar{B}$,$\bar{w}$ on ,x,y,z, axis .

The equivalent to (1) equations define Torsional-momentum $\overline{B}(B_1,B_2,B_3)$ from angular velocity $\overline{w}(w_1\ W,_2\ ,w_3)$, through the parameters J_x , J_y , J_z and through Tensor T as ,

$$T = \begin{pmatrix} J_x & -J_{xy} & -J_{zx} \\ -J_{xy} & J_y & -J_{yz} \\ -J_{zx} & -J_{yz} & J_z \end{pmatrix}$$ and (4a) becomes $\overline{B} = T\overline{w}$

Equations (4) ,(4a) give a new relation between $\overline{\rho}^{\,a}$, $\overline{B}\overline{w}$, and since $\overline{\rho}^{\,a} = x_\rho x + y_\rho y + z_\rho z$ and $\overline{B}\overline{w} = B_1w_1+B_2w_2+B_3w_3$ then $\overline{\rho}^{\,a}$ $(=J_a a^2) = J_x\ x^2+ J_y\ y^2+ J_z\ z^2 - 2J_{yz}\ yz - 2J_{zx}\ zx - 2J_{xy}\ xy$ ...(5) and $\overline{B}\overline{w}$ $(= Jw^2 = 2L) = J_x w_1^2 + J_y w_2^2 + J_z w_3^2 - 2(J_{yz}w_2w_3+J_{zx}w_2w_1+J_{xy}w_1w_2)$ ……..(5a) From equations (4a) , then (5a) becomes ,

$\frac{\partial L}{\partial w_1} = B_1$, $\frac{\partial L}{\partial w_2} = B_2$, $\frac{\partial L}{\partial w_3} = B_3$ or , $\overline{B} = \overline{\imath}\frac{\partial L}{\partial w_1} + \overline{\jmath}\frac{\partial L}{\partial w_2} + \overline{k}\frac{\partial L}{\partial w_3}$ ……..(5b)

Considering a changeable radius $\overline{a}$ and constant the product $\overline{\rho}.\overline{a} = C$ = constant then equation (5) $\overline{\rho}^{\,a}$ $(=J_a a^2) = J_x\ x^2+ J_y\ y^2+ J_z\ z^2 - 2J_{yz}\ yz - 2J_{zx}\ zx - 2J_{xy}\ xy = C$ ……..…(6)

Equation (6) defines a second degree surface , *Ellipsoid* , by the Radius-spearhead $\overline{a}$ and when , 1… From (1a) and for any radius $\overline{a} = \infty$ then $\overline{\rho} = \infty$ also , therefore $\overline{\rho}\overline{a} = \infty$ although this product was considered as constant , $\overline{\rho}\overline{a} = C$.
2… This Inertial Ellipsoid of , ⊕ sphere is referred to O center , it is a body dependent on constant C 3… The ⊕ sphere is moving , then Inertial Ellipsoid is moving also because it a body .

From equation (6) $J_a a^2 = C$ and also (Σm_i). $i^2a^2 = C$ therefore → $i = \frac{1}{a}\sqrt{\frac{C}{\Sigma m_i}}$ ……..…(7)

i.e. *the rotational radius* ,i, *on* ,a, *radius is equal to the inverse value of this radius* ,i . During displacement , $\delta\overline{a}(\delta x , \delta y , \delta z)$ on Ellipsoid , equation (6) is equal to zero so ,

$J_x x\ \delta x + J_y y\ \delta y + J_z z\ \delta z - J_{yz} y\ \delta z - J_{yz} z\ \delta y - J_{zx} z\ \delta x - J_{zx} x\ \delta z - J_{xy} x\ \delta y - J_{xy} y\ \delta x = 0$ and in case of radius $\overline{a}$ coincides with one of x,y,z, axis say ,x, then y=z=0 and , $J_x x\ \delta x - J_{xy} x\ \delta y - J_{zx} x\ \delta z = 0$ and in case of radius $\overline{a}$ coincides with one of the principal axis of Ellipsoid where then $\delta x = 0$ then $x (J_{xy} x\ \delta y + J_{xz}\ \delta z) = 0$ valuing for any variable , δy , δz and simultaneously $J_{xy}=J_{zx}= 0$.

Proceeding above logic for all principal axis then , The Centrifugal-moments are zero for any coordinate System on the Principal axis of Inertial-Ellipsoid and equations (4) become , $x_\rho = x\ .J_1$, $y_\rho = y\ .\ J_2$, $z_\rho = z\ .\ J_3$ ……..…(8) *where* , J_1 , J_2 , J_3 are the Moments of Inertia of Ellipsoid related to Principal axis , and so $\overline{\rho} = \overline{\imath}\ J_1x + \overline{\jmath}\ J_2y + \overline{k}\ J_3z$, …(8a) $\overline{\rho}^{\,a} = J_a.a^2 = J_1x^2 + J_2y^2 + J_3z^2$, ….(8b) and if a,b,c are the directional cosines of $\overline{a}$ then , $J_a = J_1\ a^2 + J_2b^2 + J_3c^2$ ….(8c) and equation of Inertia $\overline{\rho}^{\,a} = C$, becomes → $J_1x^2 + J_2y^2 + J_3z^2 = C$, $\overline{B}\overline{w} = C$ ….. (8d)

Inserting restriction $\overline{B}\overline{w} = C$ in (5a) then we have the equation ,

$J_x w_1^2 + J_y w_2^2 + J_z w_3^2 - 2(J_{yz}w_2w_3+J_{zx}w_3w_1+J_{xy}w_1w_2) = C$ ….(9)

Equation (9) defines angular velocity , $\overline{w}$ ($w_1\ W,_2\ ,w_3$) in all directions of constant, $\overline{B}\overline{w}$, *Therefore issues and for the Constant Kinetic-Energy* ,L, of ⊕ sphere $\{\overline{B}\overline{w}= 2L\}$.
Equation (9) defines the same Ellipsoid as equation (6) **, i.e.**

Every radius of Inertial-Ellipsoid acquires meter , the angular velocity which ⊕ sphere must be rotated, so that kinetic energy remains constant and = $\frac{1}{2}$ C Because of above property Inertial-Ellipsoid coincides to Angular-Velocity-Ellipsoid.

The shape of ellipsoid does not change the motion of sphere because behaves as a Rigid-body.
Considering in (4a) coordinate axis the Principal axis of Ellipsoid , then

$B_1= J_1w_1$, $B_2= J_2w_2$, $B_3= J_3w_3$, …………(10) , therefore

$\overline{B} = \overline{\imath}\ J_1w_1 + \overline{\jmath}\ J_2w_2 + \overline{k}\ J_3w_3$ and $(B^2 = J_1^2w_1^2 + J_2^2w_2^2 + J_3^2w_3^2)$ ..…(10a)

$\overline{B}\overline{w} = 2L = Jw^2 = J_1w_1^2 + J_2w_2^2 + J_3w_3^2$ ……(10b)

The equation of Ellipsoid of Angular velocity becomes , $J_1w_1^2 + J_2w_2^2 + J_3w_3^2 = C$ ......(10a)

This Changeable relation between Angular-Velocity-Ellipsoid and Rotational-Momentum as in (1a) , allows equations of motion to coincide with those of the solid ⊕ Sphere . Product $\overline{\rho}\ \overline{a}$ of (1a) variables $\overline{\rho}$ and $\overline{a}$ is constant defined in x,y,z coordinates of $\overline{a}$, and when defined in x_ρ ,y_ρ ,z_ρ coordinates by choosing Principal-axis of Inertial-Ellipsoid then $x = \frac{1}{J_1}\ x_\rho$, $y = \frac{1}{J_2}\ y_\rho$, $z = \frac{1}{J_3}\ z_\rho$, $\overline{\rho}\ \overline{a}\ (= x_\rho x + y_\rho y + z_\rho z) = \frac{1}{J_1}\ x_\rho^{\ 2} + \frac{1}{J_2}\ y_\rho^{\ 2} + \frac{1}{J_3}\ z_{\rho_2}$ and considering $\overline{\rho}\ \overline{a} = C$

$$\frac{1}{J_1}\ x_\rho^{\ 2} + \frac{1}{J_2}\ y_\rho^{\ 2} + \frac{1}{J_3}\ z_\rho^{\ 2} = C \qquad \text{……………......(11)}$$

Equation (11) consists another Ellipsoid with the same positions of Principal-axis .
The two surfaces of (8d),(11) are Interchangeable .
Considering $\overline{\rho_1}$, $\overline{a}_1$, $\overline{\rho_2}$, $\overline{a_2}$ as in (1) as radii vectors and as the equal (8a) then $\overline{\rho_1} = \overline{\imath}\ J_1x_1 + \overline{\jmath}\ J_2y_1 + \overline{k}\ J_3z_1$, $\overline{\rho_1} = \overline{\imath}\ J_1x_2 + \overline{\jmath}\ J_2y_2 + \overline{k}\ J_3z_2$ and for $\overline{a}_1 = \overline{\imath}\ x_1 + \overline{\jmath}\ y_1 + \overline{k}\ z_1$, $\overline{a_2} = \overline{\imath}\ x_2 + \overline{\jmath}\ y_2 + \overline{k}\ z_2$, then become $\overline{\rho_1}\ \overline{a_2} = J_1x_1x_2 + J_2y_1y_2 + J_3z_1z_2 = \overline{\rho_2}\ \overline{a}_1$ ……………......(12)

Considering two radii $\overline{\rho}$, $\overline{a}$ and their bordering $\overline{\rho} + \delta\overline{\rho}$, $\overline{a}+\delta\overline{a}$ with $\delta\overline{\rho}$, $\delta\overline{a}$ zero variations i.e. replacing $\bar{a}_1, \bar{a}_2$, $\bar{\rho}_1, \bar{\rho}_2$ of (12) with responding , $\bar{a}_1$, $\bar{a}+\delta\bar{a}$ and $\bar{\rho}$, $\bar{\rho}+\delta\bar{\rho}$ then $\bar{\rho}.\delta\bar{a}= \bar{a}\delta\bar{\rho}$..(12a) and because variable is not under restriction $\bar{\rho}\bar{a} = C$, then $\bar{\rho}.\delta\bar{a} + \bar{a}.\delta\bar{\rho} = 0$ ……....(12b) and from (12a),(12b) is concluded $\rightarrow \bar{\rho}\,\delta\bar{a} = 0$ and $\bar{a}\,\delta\bar{\rho} = 0$ ……...(12c)

Condition $\bar{\rho}.\bar{a}$=C defines as (1a) two surfaces : ***First-Surface*** is the end points of radii $\bar{a}$, of the Inertial Ellipsoid and the ***Second-surface*** is the end points of radii $\bar{\rho}$, for the changeable Ellipsoid as in (11). The two surfaces are joint through (12c) as in Figure.14-2.

Remarks :

1..Since radii $\bar{a}$ Surface , consist the Inertial-Ellipsoid , i.e***. the Reaction to the Angular-Velocity- Ellipsoid and which is the Mass of Space of the*** ⊕ **sphere** , so ***radii*** $\bar{\rho}$, consist the Changeable Momentum - Ellipsoid , i.e. ***the Angular-Velocity-Ellipsoid which is the Energy in Sphere.***

This Ellipsoid is not conserved when in Principal-axis even in the absence of applied torques.

2..Since in radii $\bar{a}$ Surface , of *Inertial-Ellipsoid* due to Angular-Velocity corresponds the radii $\bar{\rho}$, perpendicular to ,$\delta\bar{a}$, vectors , therefore it is a ***Tangential-Plane of*** $\bar{a}$ ***Surface*** , on spearhead of $\bar{a}$.

3..Since in radii $\bar{\rho}$ Surface , of *Energy-Ellipsoid* due to Angular-Velocity corresponds the radii $\bar{a}$, perpendicular to , $\delta\bar{\rho}$, vectors , therefore it is a ***Tangential-Plane of*** $\bar{\rho}$ ***Surface*** , on spearhead of $\bar{\rho}$. **4.**.The two Ellipsoids that of , *Angular-velocity-Ellipsoid* , and that of , *Momentum≡Energyy-Ellipsoid* are Interchangeable , meaning that Energy ≡ Momentum from Chaos , sweeps out , a cone centered on the Ecliptic-pole of Angular-velocity Ellipsoid as Spin , in this tiny Energy-ellipsoid . The Two magnitudes in the absence of Principal axis are both conserved .

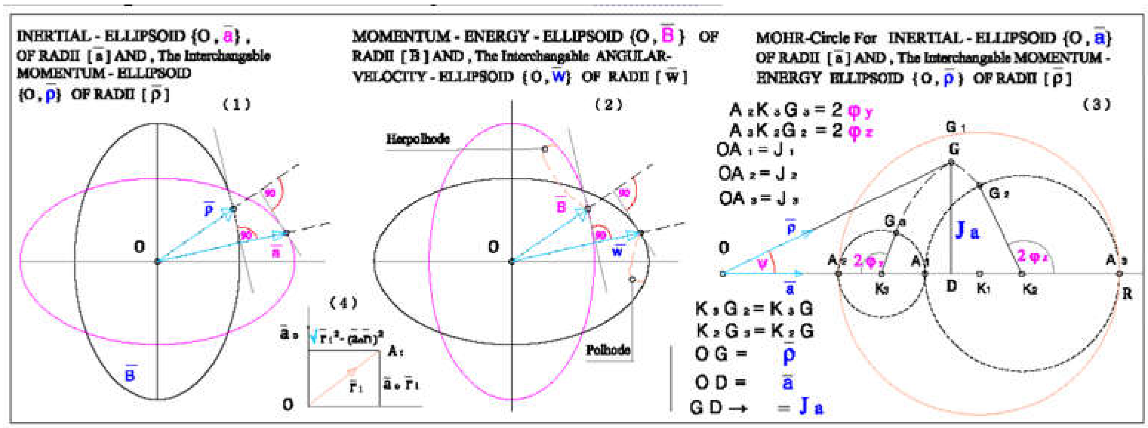


Figure.14. In (4) is shown the Geometrical-meaning of $\bar{a}_o\bar{r}_i$ and $\sqrt{r_i^{\,2} - (\bar{a}_o\,\bar{r}_i)^2}$ terms.

In (1) is shown the *Inertial - Ellipsoid* (O, $\bar{a}$) of Radii , $\bar{a}$, and the Interchangable Momentum – Ellipsoid (O , $\bar{\rho}$) of Radii , $\bar{\rho}$. (O , $\bar{\rho}$)

In (2) is shown the *Momentum - Energy - Ellipsoid* (O, $\bar{B}$) of Radii , $\bar{B}$, and the Interchangable Angular Velocity – Ellipsoid (O , $\bar{w}$) of Radii , $\bar{w}$.

In (3) is shown in Mohr-method , the Geometrical construction from the two Interchangable Ellipsoids → *The Energy – Rotational Momentum–Ellipsoid* (O , $\bar{\rho}$) , { ***Work*** } and the Angular-Velocity-vector -Unit-sphere → *The Inertial- Ellipsoid* (O , $\bar{a}$) , { ***Force*** } and the reaction to the velocity-change-motion → *The Mass - Ellipsoid* (GD=J_a≡$\bar{M}$) , { ***Mass*** } The above property of the two Interchangable-Ellipsoids defines the deep relation between ,

The Angular-velocity-Ellipsoid → $J_1w_1^2 + J_2w_2^2 + J_3w_3^2 = 2L = C$ …....(13) *and*

The Momentum-Energy-Ellipsoid → $\frac{1}{J_1}B_1^{\,2} + \frac{1}{J_2}B_2^{\,2} + \frac{1}{J_3}B_{3_2} = 2L = C$ …....(13a)

Above equation (13) Fig.14-2 define that , in radii $\bar{w}$, Angular-Velocity of ⊕ **sphere** , corresponds Radii $\bar{B}$ of (13a) Fig.14-3 defining Rotational-Angular-Momentum from the common point , O, of ⊖ ***sphere*** . Radii $\bar{B}$ is perpendicular on spearhead $\bar{w}$ tangential-Plane , of the Angular-velocity- Ellipsoid and radii $\bar{w}$, is perpendicular on spearhead $\bar{B}$ tangential-Plane , of the Rotational - Momentum-Energy –Ellipsoid as in Figure 14-3.

It was shown in ***Material-Geometry*** [58] , that Velocity-vectors and that of light–velocity becomes from geometry as *expression of Lorentz factor ,γ, from* **sec φ** = γ = OD$_A$: AD$_A$= ± 1 / [$\sqrt{1 - (v/c)^2}$].

It was accepted in (1a) that the correlated vectors , $\bar{\rho}$,$\bar{a}$ follow restriction $\bar{\rho}\bar{a}$ = constant C . From Pythagoras theorem in Euclidean-geometry the equation of Unit-Sphere in a x,y,z, coordinate System is → $a^2 = x^2+y^2+z^2 = 1$ (14) , and in vector form , $\bar{a}\bar{a} = 1$. (14a)

5.. Let see variation ,*motion*, of , $\bar{\rho}$ vector-radii , to the corresponding vector-radii $\bar{a}$,as in (1a) under Premise → the spearhead $\bar{a}$, lies on Unit-Sphere as in (14)-(14a) ←

Choosing Principal axis as the coordinate system of Inertial-Ellipsoid , equalities of (8) are ,

$x = \frac{1}{J_1} x_\rho$, $y = \frac{1}{J_2} y_\rho$, $z = \frac{1}{J_3} z_\rho$ *and from* (14) *follows* $\frac{1}{J_1^2} x_\rho^2 + \frac{1}{J_2^2} y_\rho^2 + \frac{1}{J_3^2} z_{\rho_2} = 1$ ....(15)
as equation of the surface on which , spearhead of radii $\bar{\rho}$ (x_ρ , y_ρ , z_ρ) , is displaced .
This surface is responding to (1a) Ellipsoid which semi-axis are the distances J_1 , J_2 , J_3 . The geometrical meaning of this view is seen when in (2) is placed the relation $a^2=1$ and $\bar{\rho}\bar{a} = J_a$ ***i.e.***

The Orthogonal-Projection of , $\bar{\rho}$ radii on the corresponding $\bar{a}$ radii of Unit-Sphere , provides the meter of Moment of Inertia of , Unit-Sphere **.**
Above conclusion demonstrates the method of Geometric-presentation of Sphere`s Moment of inertia to different axis through constant point O , and this because the Ellipsoid-radii -length acquires the Reciprocal meter-length of the corresponding Inertial-radii –Sphere-meter .
Mohr method impresses on Unit-Sphere , $\bar{a}$, the Projection of the Rotational-Momentum , $\bar{\rho}$, and finds the Inertial momentum J_a , i.e. in Unit-Energy-Sphere , Kinetic – Energy as Momentum , defines the Reaction to this Energy-motion .

6.. For the center , K , of ⊕ sphere , issues $\overline{v_K} = [\ \bar{w}.\overline{r_K}\ ] = [\frac{\sigma[1+\sqrt{5}]}{2r} 2r] = \sigma\,[1+\sqrt{5}]$ ....(a)
and $\bar{B} = [\bar{r}\,.\,m\bar{v}\,] = [\,rm.\ \sigma(1+\sqrt{5})]$ and for m = 1 then $\bar{B} = [\,r\,\sigma\,(1+\sqrt{5})\,]$ ....(b) Interchangable Ellipsoids of Angular velocity (13) , and Momentum (13a) for the same Moment of Inertia $J_1 = J_2 = J_3 = J$, Angular Velocity $w_1= w_2= w_3=w$, and Momentum $B_1=B_2=B_3= B$ become $3J\,w^2 = C$ and $3B^2/J = C$ and since for circle $J = \frac{\pi r^4}{4}$ then $\frac{3\pi r^4}{4} w^2 = C = (\frac{3\pi r2}{4})w^2 =$
$(\frac{3\pi r^2}{4})(rw)^2 = (\frac{3\pi r^2}{4})[\frac{\sigma}{2}(1+\sqrt{5})]^2 = \frac{3\pi r^2.\sigma^2}{8}[3+\sqrt{5}]$ → ***The Ellipsoid of Angular velocity*** $\bar{\mathbf{a}}$.......(c)
and $3B^2/J = \frac{3(rmv)^2}{J} = \frac{3(rv)^2}{J} = \frac{3r^2.\sigma^2[3+\sqrt{5}]}{2}(\frac{4}{\pi r^4}) = \frac{6.\sigma^2[3+\sqrt{5}]}{\pi r^2}$ → ***The Momentum-Ellipsoid*** $\bar{\boldsymbol{\rho}}$.......(d)
Equations (c),(d) define the two interchangeable Ellipsoids related to Sphere-radius , r , Stress , σ .

2.2. ***Mohr-circle , method*** :
1.. On OR straight-line and from initial point ,O, sectors OA_1 ,OA2 , OA_3 are taken equal to J_1 J,2 , J_3 respectively .
2.. On diameters A_2A_3 , A_3A_1 , A_1A_2 are drawn semicircles with K_1 , K_2 , K_3 centers .
3.. Let angles φ_y , φ_z , be the , $\bar{a}$ vector to , y , z , axis , and draw the circles K_3G_3 , K_2G_2 from K_3 , K_2 centers forming to K_3A_2 , K_2A_3 , angles $2\varphi_y$, $2\varphi_z$.
4.. Draw the circles G_3G , G_2G , with centers K_2 , K_3 and G their intersection .
5.. Vectors , **OG** define the Magnitude of , $\bar{\boldsymbol{\rho}}$, *Rotational -Momentum-Energy -Ellipsoid vector* , **OD** define the Magnitude of , $\bar{\mathbf{a}}$, *Angular-Velocity-Radius-spearhead-Ellipsoid vector* , with angle , ψ =GOR = GOD , between $\bar{\rho}$ and $\bar{a}$ vectors ,
GD define the Magnitude of , $\bar{\mathbf{M}} = \mathbf{J_a}$, which is *The meter of the Change = Reaction , the Orthogonal-projective of* $\bar{\rho}$ Radii to $\bar{a}$ Radii , of the Unit-sphere , and which consists the moment of inertia of Sphere , i.e. that what we call , ***mass*** , in Classical mechanics .
Remarks :
Moment of inertia , $J_{(a)}$, of a perpendicular to $\bar{a}$ Plane passing through O is used Ellipsoid (1a) from relation $\bar{\rho} = \Sigma\ (m_i\,\bar{r}_i\,.\,\bar{a}\,\bar{r}_i)$ .... (16) , and for
$\bar{\rho}\,\bar{a} = \Sigma\ m_i(\bar{a}\,\bar{r}_i\,)^2 = \bar{a}^{\,2}J_{(a)}$ and if $\bar{a}$(x ,y ,z) defines the Unit-Sphere-Radii , then $\bar{\rho}\,\bar{a} = J_{(a)}$ Equations in (4) become ,
$x_\rho = J_{(x)}\,x + J_{xy}\,y + J_{zx}\,z$, $y_\rho = J_{xy}\,x + J_{(y)}\,y + J_{yz}\,z$, $z_\rho = J_{zx}\,x + J_{yz}\,y + J_{(z)}\,z$ ..... (16a) and when defining the Principal-axis $J_{xy} = J_{yz} = J_{zx} = 0$ as coordinate axis for Ellipsoid then , $x_\rho = J_{(1)}\,x$, $y_\rho = J_{(2)}\,y$, $z_\rho = J_{(3)}\,z$ ..... (16b)
and the Perpendicular *Ellipsoid* becomes , $\frac{1}{J_{(1)}^{\ 2}}x_\rho^{\ 2} + \frac{1}{J_{(2)}^{\ 2}}y_\rho^{\ 2} + \frac{1}{J_{(3)}^{\ 2}}z_\rho^{\ 2} = 1$ ..... (16c)
and Mohr method is applied for sectors OA_1 ,OA2 , OA_3 are equal to $J_{(1)}$, $J_{(2)}$, $J_{(3)}$, which are the moments of Inertia to Principal-Planes .

1.. Angle , ψ = GOD = 0 → defines $\bar{\rho} \equiv \bar{a}$ and , $\bar{M} = J_a \equiv 0$, meaning that Energy-Momentum-Ellipsoid , $\bar{\rho}$, coincides with that of Angular-Velocity-Ellipsoid vector , $\bar{a}$, and the Velocity-Reaction-Ellipsoid $\bar{M} = J_a \equiv 0$
2.. Angle , ψ = GOD = 90° → defines $\bar{\rho} \equiv \bar{M}$ and , $\bar{a} \equiv 0$, meaning that Energy-Momentum- Ellipsoid , $\bar{\rho}$, coincides with that of Velocity- Reaction-Ellipsoid $\bar{M}$.
Since Work $\bar{W} = \bar{F}.\ \bar{ds} = (\ F.\cos\psi\ ).\ \bar{ds} = F\ .\ (\bar{ds}.\ \cos\psi)$, then Force , F , defines the ***Kinetic-Energy*** ≡ ***Energy*** , and Displacement $(\bar{ds}.\ \cos\psi)$ defines the ***Discrete-Monad*** ≡ ***Space*** which represent the two magnitudes , $\bar{\rho}$ and $\bar{a}$ i.e. → $\bar{\mathbf{W}} \equiv \bar{\boldsymbol{\rho}}$ ***and*** $(\bar{\mathbf{ds}}.\ \mathbf{cos}\ \boldsymbol{\psi}) \equiv \bar{\mathbf{a}}$ **.**
In trigonometry cos ψ = - cos(90 + ψ) = sin(90 − ψ) , so from figure , sin(90 − ψ) = GD , i.e. ***in Extrema case*** , where Space = $(\bar{ds}.\ \cos\psi)=0$, Kinetic-Energy does not vanish since then holds

$\overline{W} = \overline{F}$ = Constant = GD = J_a = *The reaction to the velocity-motion* = *The Mass-Ellipsoid* = M

3.. Angle , ψ = GOD ≇ 0 → defines $\overline{\rho} ≇ \overline{a} ≇ M ≇ 0$, meaning that exist the magnitudes ,

Rotational-Momentum Ellipsoid ≡ Work ≡ $\overline{\rho}$, → ***the Energy-vector***

Angular-Velocity–Inertial-Ellipsoid ≡ Force ≡ $\overline{a}$, → ***the Space - vector***

Reaction to velocity-change-motion ≡ Mass- scalar M ≡ J_a , → ***the Mass – meter***

It was shown in [58] *that the maximum velocity in a closed system occurs in Common circle , when the two velocities ,* $\overline{c}$ *,* $\overline{v}$ *are perpendicular between them , and not producing Work , from where then dispersion follows Pythagoras theorem and the resultant Quantized linear Space length ,r, becomes , as the Resultant of Energy Vectors ,* $r = |(\overline{c}.T)| = \sqrt{v^2 + c^2}$ *and by using Space Vector* $r = |(\overline{c}.T)| = \sqrt{v^2 + c^2}$ *then , The total Rotating energy is* →

$\pm \overline{\Lambda} = \overline{p}.r = (M.c).r = (M.c).\sqrt{v^2 + c^2}$ *and squaring both sites then ,*

$[\pm \overline{\Lambda}]^2 = p^2.r^2 = M^2.c^2.(v^2+c^2) = (M^2.v^2).c^2 + M^2.c^4 = (p^2.c^2) + M^2.c^4 = [p.c]^2 + [m_o.c^2]^2$.….(c)

*The Geometrical-analogous happens in Figure.*14.-3 *where according to Pythagoras-theorem holds ,*

$(\overline{\rho})^2 + (\overline{a})^2 = (\overline{M} = J_a)^2$ ……(d)

Equations (a) and (b) are Identical in Energy-Space content and define ,

[Work ≡ Energy ≡ Torsional-momentum] ² = [Moving-Space-Energy] ² + [Rest-Space-Energy] ².

2.3. ***Second-degree Moments in Sphere and Planes*** :

Considering Density of ⊕ sphere the same for the (i) points then mass m_i at every point is (m) for the tiny volume ,dV, and the different elements become , m.dV , and for m=1 ,dV, and for Plane ,dF. Placing x=0 in (5) and $\overline{a}$ in ,yz, Plane , then $J_a a^2 = J_y y^2 + J_z z^2 - 2J_{yz} yz$ ………….. (17) or $J_a = J_y \cos^2\varphi + J_z \sin^2\varphi - 2 J_{yz} \cos\varphi . \sin\varphi$ ………….. (17a)

where φ , is the angle of $\overline{a}$ radii to , y axis , and give the moment of inertia J_a of the Sphere to the axis on Plane through initial point , O , of rotation . In case that coordinate axis coincide with the Principal axis then equations (17) and (17a) become , ,

$J_a a^2 = J_2 y^2 + J_3 z^2$ ……….. (17b) and $J_a = J_2 \cos^2\varphi + J_3 \sin^2\varphi$ …….. (17c) Equations denote Moment of inertia for all Planes to axis on Planes , and in case of Plane surfaces the Inertia-Ellipsis which is $J_2 y^2 + J_3 z^2 = C$ ……..….. (18)

2.4. ***Euler-Lagrange , equations of motion*** :

I.. For the positioning of a rotating Solid around a Fixed-Point O , with a three coordinate system x,y,z at ,O , is chosen a second three coordinate system x`, y`, z` at , O , joint to the moving solid and rotated to O . Its position define the nine -(9) Directional-cosines of the axis i.e. the products , i`ī , j`j̄ , k`k̄ . The six identities joining cosines , degrade the six to three parameters and Solid acquires Three-degrees-of freedom around the fix point O. Euler-method-System is consisted of three parameters which are the three angles between axis .

Let be Unit-vector $\overline{s}_o$ on x,y - x`y` Planes section , such that system (k`, $\overline{k}$, $\overline{s}_o$) is right-turned .

Euler angles are in Figure.15-(1) , *Angle ,φ,* for, i` to $\overline{s}_o$ axis , *Angle ,ϑ ,* for, k` to $\overline{k}$ axis and *Angle , ψ ,* for, $\overline{s}_o$ to ī axis . Angles φ ,ϑ , ψ , follow the Right-hand-rule-direction along the axis k`, $\overline{s}_o$ ,$\overline{k}$ , of rotation respectively . Angle φ , defines the position of $\overline{s}_o$ section in , x` , y` plane . On perpendicular to $\overline{s}_o$ plane angle ,ϑ , defines the position of ,z, axis while on perpendicular to z , plane angle , ψ , defines the position of ,x, axis . In this way angles φ ,ϑ , ψ , define the position of the moving-system , x , y , z related to the fixed , x` , y` , z`.

The Directional-cosines i`ī , j`j̄ , k`k̄ ,of ī , j̄ , $\overline{k}$ axis related to i`, j`, k` axis is done by the displacement of , x , y , z system , from i`, j`, k` position to , ī , j̄ , $\overline{k}$ , in three stages as , 1.. By rotating on k` axis , according to , φ , angle , such that ,x, axis moves from i` to the $\overline{s}_o$ position where then issue as in F.15- (2) the equalities ,

$\overline{s}_o$ = i`.cos φ + j`. sin φ and $\overline{q}_o$ = - i`.sin φ + j`. cos φ ……..…….. (a) where $\overline{q}_o$ index is perpendicular to k`, and { $\overline{s}_o$ and $\overline{q}_o$ } , { k` and $\overline{s}_o$ } , Right-hand-rule direction system .

2.. By rotating on $\overline{s}_o$ axis , according to , ϑ , angle , such that ,z, axis moves from k` to the $\overline{k}$ position where then issue as in F.15- (3) the equalities , $\overline{t}_o$ = k`.sin ϑ + $\overline{q}_o$.cos ϑ and $\overline{k}$ = k`.cos ϑ - $\overline{q}_o$.sin ϑ ……..…….. (b) where $\overline{t}_o$ index is perpendicular to $\overline{k}$, and { $\overline{s}_o$ and $\overline{t}_o$ } , { $\overline{k}$ and $\overline{s}_o$ } , Right-hand-rule direction system .

3.. By rotating on $\overline{k}$ axis , according to , ψ, angle , such that ,x, axis moves from $\overline{s}_o$ to the ī position where then issue as in F.15- (4) the equalities ,

ī = $\overline{s}_o$. cos ψ + $\overline{t}_o$ sin ψ and j̄ = - $\overline{s}_o$. sin ψ + $\overline{t}_o$ sin ψ ……..…….. (c)

Thus axis ,z, and ,x, arrive to { $\overline{k}$, ī } final positions carrying the (x,y,z) Solid to (ī , j̄ , $\overline{k}$) Position , and if between equalities (a) (b) (c) delete Unit-vectors $\overline{s}_o$, $\overline{q}_o$, $\overline{t}_o$, by placing in (b) , the ($\overline{q}_o$ } of (a) and then in (c) the { $\overline{s}_o$ and $\overline{t}_o$ } of (a) (b) , then acquire expression of equations in ī,j̄, $\overline{k}$ from those of i`, j`, k` *and* of angles , φ ,ϑ , ψ . The Directional-cosines are

$i\grave{\bar{\imath}}$ = cos a_1 == cos φ.cos ψ - sin φ. cos ϑ.sin ψ $i\grave{\bar{\jmath}}$ = cos a_2 == - cos φ.sin ψ - sin φ. cos ϑ.cos ψ $\bar{\imath}\grave{\bar{k}}$ = cos a_3 == sin φ.sin ϑ.

$j\grave{\bar{\imath}}$ = cos b_1 == sin φ.cos ψ + cos φ. cos ϑ.sin ψ ……….….. (d) $j\grave{\bar{\jmath}}$ = cos b_2 == - sin φ.sin ψ + cos φ. cos ϑ.cos ψ

$\bar{\jmath}\grave{\bar{k}}$ = cos b_3 == - cos φ. sin ϑ

$\bar{k}\grave{\bar{\imath}}$ = cos c_1 == sin ϑ. sin ψ

$\bar{k}\grave{\bar{\jmath}}$ = cos c_2 == sin ϑ. cos ψ

$\bar{k}\grave{\bar{k}}$ = cos c_3 == cos ϑ

II

.. For the rotation of a Solid with ,w, angular velocity is used the following equation ,

$$\bar{w} = \bar{k}\grave{} \frac{d\varphi}{dt} + \bar{s}_o \frac{d\vartheta}{dt} + \bar{k} \frac{d\psi}{dt} . \quad \text{……..….. (19)}$$

composed of ***one*** Rotation , *around* , z`, *axis with angular velocity* $\frac{d\varphi}{dt}$, ***a second*** , *around* , $\bar{s}_o$, *axis with angular velocity* $\frac{d\vartheta}{dt}$, and ***a third*** , *around* , z , *axis with angular velocity* $\frac{d\psi}{dt}$.

Since is needed Angular-velocity , $\bar{w}$, tobe related to the Fix to Solid directions of x , y , z , axis then the three components w_1 , w_2 , w_3 of vector $\bar{w} = \bar{\imath}\, w_1 + \bar{\jmath}\, w_2 + \bar{k}\, w_3$ are related to angles , φ ,ϑ , ψ , and to angular velocities $\frac{d\varphi}{dt}$, $\frac{d\vartheta}{dt}$, $\frac{d\psi}{dt}$.

Projecting (19) on $\bar{\imath}, \bar{\jmath}, \bar{k}$ axis then become the equations ,

$$w_1 = \bar{w}.\bar{\imath} = \frac{d\varphi}{dt}.\overline{k\grave{}\imath} + \frac{d\vartheta}{dt}.\overline{s_o\imath} + \frac{d\psi}{dt}.\overline{k\imath}$$
$$w_2 = \bar{w}.\bar{\jmath} = \frac{d\varphi}{dt}.\overline{k\grave{}\jmath} + \frac{d\vartheta}{dt}.\overline{s_o\jmath} + \frac{d\psi}{dt}.\overline{k\jmath} \quad \text{……….. (19a)}$$
$$w_3 = \bar{w}.\bar{k} = \frac{d\varphi}{dt}.\overline{k\grave{}k} + \frac{d\vartheta}{dt}.\overline{s_o k} + \frac{d\psi}{dt}.\overline{kk}$$

and since holds , $k\bar{k} = 1$, $k\bar{\imath} = k\bar{\jmath} = k\bar{s}_o = 0$, $\bar{s}_o.\bar{\imath}$ = cos ψ , $\bar{s}_o.\bar{\jmath}$ = - sin ψ then from (d) exists $\bar{k}\grave{\bar{\imath}}$ == sin ϑ sin ψ , $\bar{k}\grave{\bar{\jmath}}$ = sin ϑ cos ψ , $\bar{k}\grave{\bar{k}}$ =cos ϑ and replacing above to (19a) then become Euler equations ,for Angular-Velocity-components ,

$$w_1 = \frac{d\varphi}{dt} \sin\vartheta \sin\psi + \frac{d\vartheta}{dt} \cos\psi$$
$$w_2 = \frac{d\varphi}{dt} \sin\vartheta \cos\psi - \frac{d\vartheta}{dt} \sin\psi \quad \text{……….. (20)}$$
$$w_3 = \frac{d\varphi}{dt} \cos\vartheta + \frac{d\psi}{dt}$$

which are related to the three angles , φ ,ϑ , ψ of rotation.

III. Euler equations , for Angular-Velocity $\bar{w}$ and Momentum $\bar{B}$:

In (10)-(10a) Angular-velocity-Ellipsoid and Momentum $\bar{B}$ are simplified when defined by the projections of, w_1 , w_2 , w_3 and B_1 , B_2 , B_3 as ,

$B_1 = J_1w_1$, $B_2 = J_2w_2$, $B_3 = J_3w_3$,

$\bar{B} = J_1w_1\bar{\imath} + J_2w_2\bar{\jmath} + J_3w_3\,\bar{k}$ ……….. (10)

$B^2 = J_1^2w_1^2 + J_2^2w_2^2 + J_3^2w_3^{\,2}$ ……….. (10a)

$\bar{B}\bar{w} = Jw^2 (= 2L) = J_1^{\,2}w_1^{\,2} + J_2^{\,2}w_2^{\,2} + J_3^{\,2}w_3^{\,2}$ …..….. (10b) Placing Momentum-equation (10a) in equation $\frac{d\bar{B}}{dt} = \bar{M} = \bar{\imath}\ M_1 + \bar{\jmath}\ M_2 + \bar{k}\ M_3$ then ,

$$\bar{\imath} J_1 \frac{dw_1}{dt} + \bar{\imath} J_2 \frac{dw_2}{dt} + \bar{\imath} J_3 \frac{dw_3}{dt} + \frac{d\bar{\imath}}{dt} J_1 w_1 + \frac{d\bar{\jmath}}{dt} J_2 w_2 + \frac{d\bar{k}}{dt} J_3 w_3 = \bar{\imath}\ M_1 + \bar{\jmath}\ M_2 + \bar{k}\ M_3$$

= $\bar{M} = \bar{\imath}\,M_1 + \bar{\jmath}\,M_2 + \bar{k}\,M_3$ where M_1 , M_2 , M_3 are the Momentum-components .

Since from vector-calculus ,

$\frac{d\bar{\imath}}{dt} = [\,\bar{w}\,\bar{\imath}\,] = \bar{\jmath}w_3 - \bar{k}w_3$, $\frac{d\bar{\jmath}}{dt} = [\,\bar{w}\,\bar{\jmath}\,] = \bar{k}w_1 - \bar{\imath}w_3$, $\frac{d\bar{k}}{dt} = [\,\bar{w}\,\bar{k}\,] = \bar{\imath}w_2 - \bar{\jmath}w_1$, then

$$J_1 \frac{dw_1}{dt} - (J_2 - J_3).\ w_2w_3 = M_1$$
$$J_2 \frac{dw_2}{dt} - (J_3 - J_1).\ w_3w_1 = M_2 \quad \text{……….. (21)}$$
$$J_3 \frac{dw_3}{dt} - (J_1 - J_2).\ w_1w_2 = M_3$$

Equations (21) are differential equations of the first order when external moments are given related to time and define ***Angular-velocity-components*** { w_1 , w_2 , w_3 } and the ***Position*** { φ ,ϑ , ψ } from (20) . If moment $\bar{M}$ is given related to position only , then w_1 , w_2 , w_3 } are placed in (21) where then position is produced by differential equations of second order .

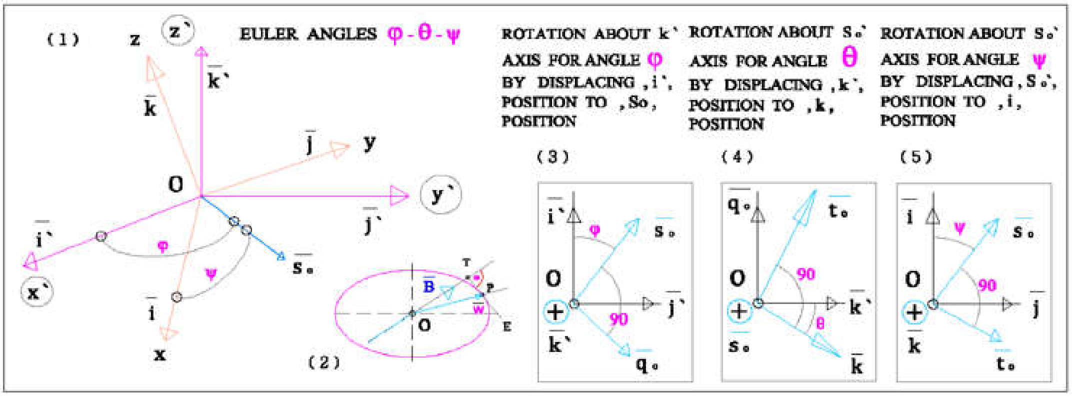


Figure.16. In (1) are shown Euler angles { φ ,ϑ , ψ } in a *Fix*-Three-Coordinate-System x , y , z through ,O, and one *Movable* x`, y`, z` at , O .
In (2) is shown the Spearhead , P , of OP Angular-velocity $\bar{w}$ vector , and the

Tangzential Plane ,E ,to Angular-velocity-Ellipsoid $\bar{w}$, on Momentum vector OT = $\frac{2L}{B}$ In (3) , (4) , (5) are shown the Three-Stages nedded for transforming $\bar{i}$`, $\bar{j}$`, $\bar{k}$`, axis to the $\bar{i}$, $\bar{j}$, $\bar{k}$ axis .

IV. Zero Static - Moment :

For $\bar{M}$=0 i.e. the Solid is supported through center of mass and then Euler equations become ,

$$J_1\frac{dw_1}{dt} - (J_2 - J_3).\ w_2w_3 = M_1 = 0$$
$$J_2\frac{dw_2}{dt} - (J_3 - J_1).\ w_3w_1 = M_2 = 0 \qquad \text{……….. (21a)}$$
$$J_3\frac{dw_3}{dt} - (J_1 - J_2).\ w_1w_2 = M_3 = 0$$

and for rotation through Principal axis of Inertial-Ellipsoid [for ,z, axis $w_1 = w_2 = 0$] then w_1 =constant (= 0) w_2 =constant (= 0) w_3= constant

i.e. rotation is continued trough this axis with constant angular velocity $\bar{w} = \frac{\bar{v}}{r} = \frac{\sigma}{2r}[1+\sqrt{5}]$ ***of that of material-point and the three axis are called Free-axis*** .

V.. The Poinsot`s Geometrical-motion :

I.. Multiplying the first equation of (21a) by w_1 , the second by w_2 , and the third by w_3 , and adding each other then , Static-Moment $\bar{M}$ is ,

$$J_1w_1\frac{dw_1}{dt} + J_2\,w_2\frac{dw_2}{dt} + J_3w_3\frac{dw_3}{dt} = 0$$

$$\frac{1}{2}J_1{w_1}^2 + \frac{1}{2}J_2{w_2}^2 + \frac{1}{2}J_3{w_3}^2 = \text{constant} \qquad \text{……….. (21b)}$$

i.e. for $\bar{M}$ = 0 , kinetic energy $\frac{\partial B}{dt}$= constant , as in (10b).

Also multiplying the first equation of (21a) by J_1w_1 , the second by J_2w_2 , and the third by J_3w_3 , and adding each other then , Rotational-Moment $\bar{B}$ is ,

$$J_1{}^2w_1\frac{dw_1}{dt} + J_2{}^2w_2\frac{dw_2}{dt} + J_3{}^2w_3\frac{dw_3}{dt} = 0$$

$J_1^2w_1 + J_2{}^2w_2 + J_3^2w_3$ = constant ……….. (21c) i.e. for $\bar{M}$ = 0 , Rotational-momentum $\bar{B}$ = constant , as in (10b).

As Kinetic energy cannot be changed , the same also for Momentum $\bar{B}$ which is constant . The Tangential Plane E , of Angular-velocity-Ellipsoid $\bar{w}$ Spearhead point P , on Momentum vector OT , remains unmoving and this because ,

a.. Plane E , is perpendicular to the unmovable Momentum-vector $\bar{B}$,

b.. Cuts the constant sector OT = $\bar{w}\bar{b}o$, where $\bar{b}o$ is the Unit-vector on momentum $\bar{B}$,

and this because $\bar{w}\bar{b}o = \frac{\bar{w}\bar{B}}{B} = \frac{2L}{B}$ = constant , therefore

Angular-velocity-Ellipsoid ***is Rolling on the unmovable plane*** **E** , *the point of conduct* P *lyies on* OP *axis , has zero velocity , and aquires* OT *distance such Geometry positions from* O *for the unmovable plane* E , *to remain unchanged* , always by following the Solid`s-motion .
Some points of the moving Ellipsoid , which are common to unmovable E , plane , define the each one rotating axis , and are those which are of equal distance Tangential-Planes from center O .
This geometrical locus on Ellipsoid is called *Polar-axis* , ***the Polhode*** , while on Plane E , the *Unti-Polar-axis ,* ***the Herpolhode*** *,* ***on where Vector-radius*** $\bar{\mathbf{w}}$ is tracing the , *Polar-axis* on the moving-solid and the , *Unti-Polar-axis* on the Fix-system .

Remarks :
Applying above to Material-Points [⊕↔⊖] of Figure.12 then all referred are becoming ,and Since , between the two consituents , exists only Pressure , σ , which is turned to velocity , $\bar{v}$, no other External forces exist to create any Moment to the initial point O of rotation . ***Because of Zero-Moment*** , ***the motion of the*** , ⊕ , ***Content*** , ***is the only motion ,*** w ***, and it is the Rolling of the Angular-Velocity-Ellipsoid , on Plane , E ,*** (*the Polar-axis* on the *Unti-Polar-axis or the , Polar-Plane on the Unti-Polar plane*) , *Polhode on Herpolhode* , *where* , ***Momentum-Ellipsoid , $\bar{\mathbf{B}}$,*** is perpendicular to ***, Angular -velocity-Ellipsoid , $\bar{\mathbf{w}}$,*** which Planes are both circles and the , ***Unmovable plane E*** *,* Tangential to the each one circle through the three axis , ***and Parallel to Principal –axis-Plane of Ellipsoid .***
From Euler-Lagrange Mechanics and Vector analysis , ***Position vector , $\bar{\mathbf{r}}$***, always points radially from the origin O .
Velocity vector , $\bar{\mathbf{v}}$, always tangent to the path , direction , of motion .
Acceleration vector , $\bar{\mathbf{a}}$, not parallel to the radial motion but offset by the angular and Coriolis accelerations , nor tangent to the Path but offset by centripetal and radial acceleration .
II. Above analysis was presented by by Poinsot without being sufficient for the complete motion description , because does not define the positions of Solid relating to time . On Angular-velocity-Ellipsoid exists , $J_1 w_1^2 + J_2 w_2^2 + J_3 w_3^2 = 2L$ (L = constant) and Polar position is defined from relation $J_1^2 w_1^2 + J_2^2 w_2^2 + J_3^2 w_3^2 = B^2$, where , B , is the constant Momentum value .

On a constant Angular-velocity-Ellipsoid , by changing Momentum , is possible of infinite Polar-Paths according to the different motions of the Solid . Paths near maximum or minimum Principal axis become ring-shaped while near the center of axis extend to a couple of ellipses.
Rotation happens near maximum or minimum axis , continuing motion about these , and round Instaneous axis of the Polar-path , in contrary to the middle axis which are not continued , but rounded to axis off Polar-path around Ellipsoid . A complete identity of the rotated axis and the Principal axis doesn`t happen as this happens on Earth .

A.. The Integration of (21a) differential equations results to Elliptic-functions , except that of Inertial-moment- Ellipsoid and which becomes of rotation . In case of Material-Point both Momentum and Angular velocity are constants for all motions . Placing ,z, axis as the rotating then $J_1 = J_2$ and equations are simplified as ,

$$\frac{dw_1}{dt} = \frac{J_1 - J_3}{J_1} w_1 w_2 \ , \quad \frac{dw_2}{dt} = \frac{J_3 - J_1}{J_1} w_3 w_1 \ , \quad \frac{dw_3}{dt} = 0 \ , \qquad \text{………..(21d)}$$

And placing $\frac{J_1 - J_3}{J_1} = \gamma$, then $\frac{dw_1}{dt} = \gamma\, w_2 w_3 \ , \quad \frac{dw_2}{dt} = -\gamma\, w_3 w_1 \ , \quad \frac{dw_3}{dt} = 0$ ……..…(22)
From the first becomes w_3= constant , while by multiplying w_1 with its components and w_2 for the second one and adding , then $w_1 \frac{dw_1}{dt} + w_2 \frac{dw_2}{dt} = 0$ therefore , $w_1^2 + J_2 w_2^2 = a^2$ and a = the constant of integration , i.e. during motion Angular velocity value is constant unchanged and equal to $w^2 = a^2 + w_3^2$, becoming also from Poinsot`s solution . In reality since Angular velocity-Ellipsoid is symmetrical to ,**z**, axis , all of equal distance from , O, tangential planes , are symmetrically placed around ,**z**, axis and so the Polar-Paths are parallel circles .

The algebraic value of angular-velocity-vector remains unaltered and thus , ***is drawing*** *the Solid regular-Cone as* ***Polar-surface*** *around symmetrical* , z , *axis* , and in Fixed system at O , *on the unmovable Momentum-vector another Solid-regular-Cone , as* ***Anti-Polar-surface*** . Since $w_1^2 + w_2^2 = a^2$, *a circle* , by introducing parameter ,u, then is holding ,
$w_1 = a \sin u$, $w_2 = a \cos u$ and the two first equations of (22) are satisfied *when* $\frac{du}{dt} = \gamma . w_3$ and also since w_3 = constant , *when* $u = \gamma . w_3 t + b$ (where ,b, is the constant of integration), i.e. equations (22) are solved by the new system ,

$$w_1 = a \sin(w_3 t + b) \ , \quad w_2 = a \cos(w_3 t + b) \ , \quad w_3 = \text{constant} \qquad \text{..………(22a)}$$

Angular-velocity-vector is rotating the Cone-Polar-surface , and returning to initial position

$$\text{in period } T = \frac{2\pi}{w} \ , \text{ or } \ T = \left[\frac{2\pi}{\gamma w_3} \right] = \left[\frac{2\pi J_1}{w_3 (J_1 - J_2)} \right] \qquad \text{..………(22b)}$$

B.. Integrating equations (21a) became the projections of angular velocity w_1 , w_2 , w_3 related to time ,**t** . Integrating (20) on unmovable axis ,**z`,** of Momentum , then on symmetrical axis ,**z**, projection of B_3 is according to (10) , equal to $B_3 = J_3 w_3$, and for Euler ,ϑ , angle exists , $\cos\vartheta = \frac{J_3 w_3}{B}$ = constant , and ϑ = constant and Euler equations (20) become ,

$$w_1 = \frac{d\varphi}{dt}\sin\vartheta\sin\psi \quad , \quad w_2 = \frac{d\varphi}{dt}\sin\vartheta\cos\psi \qquad \text{............} (23)$$

$$w_3 = \frac{d\varphi}{dt}\cos\vartheta + \frac{d\psi}{dt}$$

From the first two exists $\frac{w_1}{w_2} = \tan\psi$ while from (22a) $\frac{w_1}{w_2} = \tan(\gamma\, w_3 t + b)$ so ,

$\psi = \gamma\, w_3 t + [\, b + k\pi\,] = \gamma\, w_3 t + \psi_o$, ψ_o= constant , and $\frac{d\psi}{dt} = \gamma\, w_3$,

Placing above in third equation of (23) then ,

$w_3 = \frac{d\varphi}{dt}\cos\vartheta + \gamma\, w_3$, or $w_3 = \frac{d\varphi}{dt}\frac{J_3 w_3}{B} + \gamma\, w_3$, therefore $\frac{d\varphi}{dt} = [1-\gamma]\,\frac{B}{J_3} = \frac{B}{J_1}$, and

since by definition of ,γ, then , $J_3 = [1-\gamma]\, J_1$ and $\varphi = \frac{B}{J_1}\, t + \varphi_o$ where φ_o is the integration constant , and the Position of the Solid is defined by Euler φ ,ϑ , ψ , angles as ,

$$\varphi = \frac{B}{J_1}\, t + \varphi_o$$
$$\vartheta = \text{constant} \qquad \text{............} (23a)$$
$$\psi = \frac{J_1 - J_3}{J_1}\, w_3\, t + \psi_o$$

2.4.1 ***Application to Material-Points*** [⊕↔⊖] of Figure.17 , and by considering Positive Constituent with angular velocity $\overline{w} = \overline{v}/r = \frac{\sigma}{2r}[1+\sqrt{5}]$ and an angle 45° from ,**x** , axis.

The Ellipsoid of angular velocity is perpendicular to the plane of motion . Moment of Inertia to ,**z**, axis is that of sphere equal to $J_3 = \frac{\pi r^4}{4}$ which is the same in all Principal axes , and

$J = J_1 = J_2 = J_3 = \frac{\pi r^4}{4}$, therefore (13) which is Angular-kinetic–energy ≡ Angular-velocity-Ellipsoid then becomes , $J_1 w_1^2 + J_2 w_2^2 + J_3 w_3^2 = 2L$, or →

$$w_1^{\,2} + w_2^{\,2} + w_3^{\,2} = \frac{2L}{J} = \frac{8L}{\pi r^4} = B^2 = 3Jw^2$$ and from (10) then $\overline{B} = [\, r\,\sigma\,(1+\sqrt{5})\,]$.

The value of constant momentum is $B^2 = 2L = 3Jw^2$, and is

$$B^2 = 3\,\frac{\pi r^4}{4}\Big(\frac{\sigma}{2r}[\,1+\sqrt{5}\,]\Big)^2 = \frac{3\pi r^2\sigma^2}{16}[\,6+2\sqrt{5}\,] = \frac{3\pi r^2\sigma^2}{8}[\,3+\sqrt{5}\,] .$$

For Planck`s length $r = 4{,}453.10^{-35}$, $r^2 = 19{,}629.10^{-70}$ and velocity $|\,\overline{v}\,| = \frac{\sigma}{r}[1+\sqrt{5}]$ becomes

$|\,\overline{v}\,| = \frac{3{,}236\,.\,\sigma}{4{,}453.10^{-35}}$, or $|\,\overline{\mathbf{v}}\,| = \overrightarrow{\mathbf{0{,}7267.10^{35}}}\,.\,\boldsymbol{\sigma}$ ....(a) and for maximum velocity $c = 3.10^8$ m/s then $\boldsymbol{\sigma} = \frac{3.10^8}{0{,}7267.10^{35}} = 4{,}128.\,10^{-27}$ Kg/m² and the value of the Angular-velocity is , $|w| = \frac{\sigma}{2r}[\,1+\sqrt{5}\,] = \Big(\frac{4{,}128.10^{-27}}{2.4.45310^{-35}}\Big)3{,}2360675 = 1{,}499\,.\,10^8$, or $|\mathbf{w}| = 1{,}5\,.\,10^8$ rad/sec and the constant figure of the Torsional-Momentum becomes ,

$B^2 = \frac{3\pi r^2\sigma^2}{8}[3+\sqrt{5}]\ \frac{3\pi 19{,}825.10^{-70}\sigma^2}{8}[5{,}236] = 122{,}315.\sigma^2$, therefore , $\mathbf{B = 11{,}06\,.\,\boldsymbol{\sigma}}$ ....(b)

and for $\sigma = 4{,}128.\,10^{-27}$ Kg/m² then $\mathbf{B} = 11{,}06\,.\,4{,}128.\,10^{-27} = 4{,}566\,.\,10^{-27}$ Kgm²/s

Constant plane ,E, is tangential to Ellipsoid at Spearhead point ,P, of $\overline{w}$ radii and Polar-line is parallel circle PT , Anti-Polar-line PS circle on plane ,E, with circle that of vector-radii projection point of Momentum $\overline{B}$. Polar-cone POT is rolling on the Fix Anti-Polar-Cone POS with constant velocity and each common line-vector the Instaneous rotating axis

. Since Angular-velocity-vector is constant then returns to initial position after the period of time $T = \left[\dfrac{2\pi J_1}{w_3(J_1 - J_2)}\right] = \left[\dfrac{2\pi}{w_3}\right]$. and for $r = 4{,}453.10^{-35}$ then , period $T = \dfrac{2\pi}{w} = \dfrac{2\pi r}{v} = \dfrac{4\pi r}{\sigma(1+\sqrt{5})} \rightarrow$

$$T = \frac{4\pi.4{,}45310^{-35}}{\sigma(1+\sqrt{5})} = \frac{1{,}729}{\sigma}.10^{-34} = \left\{\frac{1{,}729}{\sigma}\right\}.10^{-34}$$, or $\mathbf{T} = \left[\dfrac{1{,}729}{10^{34}.\sigma}\right]$ …….. (c)

and for $\sigma = 4{,}128.\ 10^{-27}$ Kg/m² , then $\mathbf{T} = \dfrac{1{,}729}{10^{34}.4{,}12810^{-27}} = \dfrac{4{,}188}{10^{8}} = 4{,}188\ .\ 10^{-8}$ s

In case of Rotation near the Symmetrical-Ellipsoid-axis , is that of *Nutation* , which happens to Earth`s rotating axis in case that doesn`t coincide with the Major-Geodic free axis .

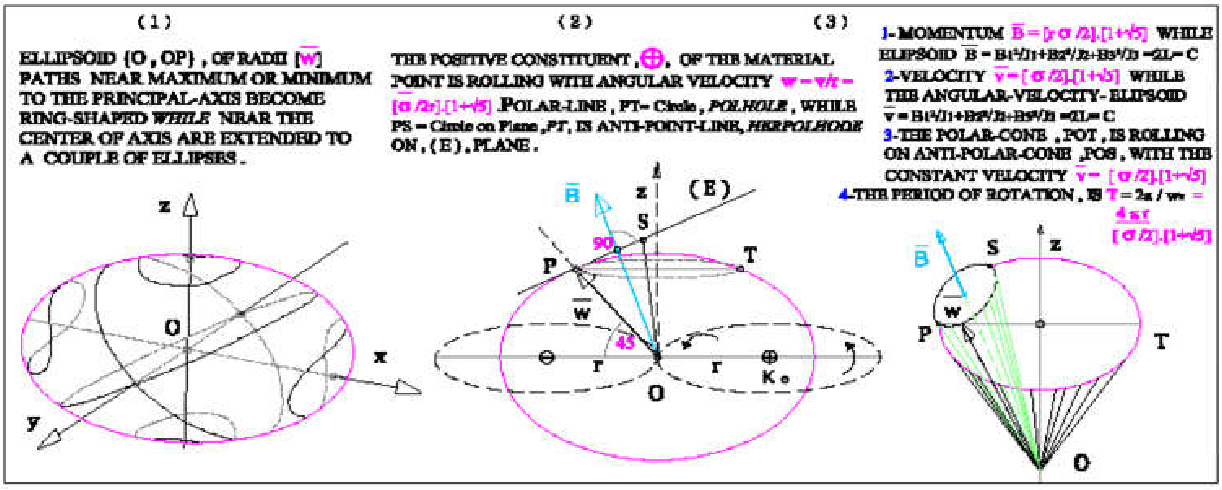


Figure.17.. In (1) , are shown Paths near maximum or minimum to Principal axis from where become-ring shaped or Small-circles , while near the center of axis extend to a couple of ellipses or Great circles .
In (2) is shown Rotation with angular-velocity $\bar{w}$, 45° to the instaneous material axis OP ≡ [⊕↔⊖] of rotation where point P is the , *Nib* ,of Angular-velocity vector $\bar{w}$ and Sweeps-Out at OP slant height of the Central , POT-Cone , with $\bar{W} = \dfrac{v}{R}$. |
In (3) are shown Polhode , *circle PT* , Herpolhode , *PS circle in plane E* . Polhode-Cone POT is rolling on the Fixed- Herpolhode-Cone POS , with the constant velocity $|\bar{v}| = \frac{\sigma}{r}[1+\sqrt{5}]$ dependent on Pressure ,σ, of the two material constituents .

3.. The Central Axial-Ellipsoid and the ⊕ constituent Rotating through constant point O .

Integrated Equations of motion become not from center of mass K_o , but from center O. Since motion of ⊕ constituent becomes from Stress ,**σ**, only , the constant coordinate system is taken at O with , **z`** , axis perpendicular to Unit-vector **k¯ ,** on $OK_o$ axis , the **x** axis . The x`, y` axis are perpendicular to $\overrightarrow{k`}$, vector and , x , y axis are perpendicular to $\overrightarrow{OK_o}$ direction. Let be $\bar{s}_o$ the common axis of , x,y and , x`, y` planes such that coordinate system k¯, $\bar{k}$, $\bar{s}_o$ Is Right-handled . Vectors $\bar{s}_o$, $\bar{t}_o$ and x , y axis , are directed to Ellipsoid-equator of Angular velocity and thus , to both ,x , y axis , moment of inertia is the same as J_1 and J_2 to z . For the definition of , ϑ , φ , ψ , related to time is proved that are needed three differential equations , the *first* , due to the intersection of , z , axis by $\overrightarrow{OK_o}$, where $M_s = 0$. Euler third equation (21) is then simplified to , $\dfrac{dw_3}{dt} = 0$ and $J_1 = J_2$ where is shown that projection of angular velocity w_3 on symmetrical axis ,z, is constant . Euler`s third equation of (20) becomes

$$(w_3 =)\ \frac{d\varphi}{dt}\cos\vartheta + \frac{d\psi}{dt} = \text{constant} \quad …………….(24)$$

A different ascertain was defined before for Moment , of the first integral which is zero on z` axis , therefore Momentum B¯k¯to this axis is constant . Momentum in the three directions $\bar{s}_o$, $\bar{t}_o$, $\bar{k}$, and to , O , axis is defined by equality , $\bar{B} = \bar{s}\,B_{os} + \bar{t}_o B_t + \bar{k}\,B_3$

$B_s = J_1 .w_s$, $B_t = J_1 .w_t$, $B_3 = J_3 .w_3$ ………….(24a)

Where , B_s , B_t , B_3 and w_s , w_t , w_3 are the corresponding projections of Momentum and Angular-velocity to the three axis . From above issues ,

→ B¯k¯= $\bar{s}$ k`$_o$¯. $J_1 w_s$ + $\bar{t}_o$k¯. $J_1 w_t$ + $\bar{k}$ k¯. $J_3 .w_3$ and since ,

$\bar{s}$ k`$_o$ = 0 , $\bar{t}_o$k¯= 0 , $\bar{k}$ k¯= cos ϑ , then → B¯k¯= $J_1 w_1 \sin\vartheta + J_3 w_3 \cos\vartheta$

From (19) issues $\bar{w} = \bar{k}' \frac{d\varphi}{dt} + \bar{s}_o \frac{d\vartheta}{dt} + \bar{k} \frac{d\psi}{dt}$ therefore ,

$$w_t = \bar{w}\bar{t}_o = \bar{k}'\bar{t}_o . \frac{d\varphi}{dt} + \bar{s}_o\bar{t}_o . \frac{d\vartheta}{dt} + \bar{k}\bar{t}_o . \frac{d\psi}{dt}$$ and since ,

$k'\bar{t}_o = \sin\vartheta$, $\bar{s}\, t_{o\,o} = 0$, $+\bar{k}\bar{t}_o = 0$, therefore , $w_t = \frac{d\varphi}{dt} \sin\vartheta$ and above become,

$(\bar{B}\,\bar{k}' =)\ J_1 \frac{d\varphi}{dt} \sin^2\vartheta + J_3\, w_3 \cos\vartheta = C_1$ where C_1 = constant …………..(24b) which is a differential equation of the first order .

Another one integral exist from velocity constancy , where then Mechanical energy remains unchanged and Dynamic energy is , V = Q(s) cos ϑ where (s) = OS , and from Kinetic-energy (L) to directions $\bar{s}_o$, $\bar{t}_o$, $\bar{k}$, on the three principal axis is ,

$L = \frac{1}{2} (J_1 .w_s^{\,2} + J_1 .w_t^{\,2} + J_3 .w_3^{\,2})$ and using equation (19)

$$w_s = \bar{k}'\bar{s}_o . \frac{d\varphi}{dt} + \bar{s}_o\bar{s}_o . \frac{d\vartheta}{dt} + \bar{k}\bar{s}_o . \frac{d\psi}{dt}$$ and since also

$k'\bar{s}_o = 0$, $\bar{s}\, s_o\, \bar{}_o = 1$, $\bar{k}\bar{s}_o = 0 \rightarrow w_s = \frac{d\vartheta}{dt}$ therefore ,

$$L + V = \frac{1}{2} J_1 [(\frac{d\vartheta}{dt})^2 + (\frac{d\varphi}{dt})^2 + \sin^2\vartheta] + \frac{1}{2} J_3\, w_{3_2} + Qs \cos\vartheta = C_2 \ …….(24c)$$

where C_2= constant , therefore the system of the three differential equations is , $\dot{\varphi} \cos\vartheta + \dot{\psi} = w_3$,

$J_1\dot{\varphi} \sin^2\vartheta + J_3\, w_3 \cos\vartheta = C_1$ ……………………(24d)

$\frac{1}{2} J_1 (\dot{\vartheta}^2 + \dot{\varphi}^2) + \frac{1}{2} J_3 w_3 + Qs \cos\vartheta) = C_2$ where , w_3 , C_1 , C_2

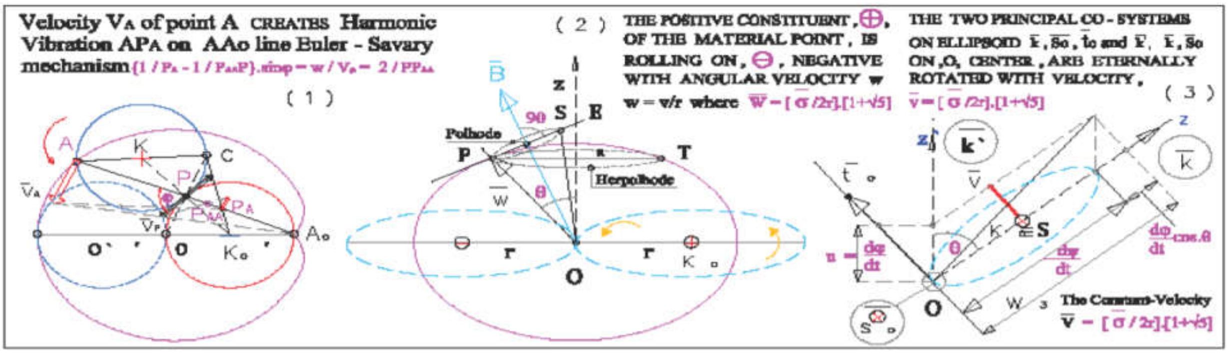


Figure.18. In (**1**) ***Material point AP*** , {the two circles ⊖ ≡ K_o,KoP – ⊕ ≡ K , KP } of the Space point A (+) and Anti-point P(-) is rotating through point A_o , which is the center of Common circle and form material angle $\vartheta = \vartheta_A.t = (\frac{v_A}{\sqrt{c^2-r^2}})t$, CREATE the Cardioid Envelope curves generated by the above Vibrating -Velocity-Energy-Geometry-Segment , A P_A , on , AA_o , rotating line.

In (**2**) is shown Angular-velocity-vector $\bar{w}$ Ellipsoid , and $\bar{B}$ Momentum-Ellipsoid both

Due to the Opposite-Stresses , ±σ , which create the constant velocity $|\bar{v}| = \frac{\sigma}{2r} [1+ \sqrt{5}]$ of the K_o Rotating-Center of mass of the ⊕ constituent .

In (**3**) are shown the two Coordinate -Systems on Principal axis , the one of the ,⊕, Central-Momentum-Ellipsoid about the Fixed center ,**O** , {the $\bar{s}_o$, $\bar{t}_o$, $\bar{k}$, System } of the POT Cone and the other one of the same ,⊕, rotated constituent , around the Instaneous axis of rotation , **z** , of POS-Cone and about center ,**O**, {the $\bar{s}_o$, $\bar{k}'$, $\bar{k}$, System } . According to Poinsot , *on Symmetrical-Ellipsoid around an axis* , the equidistance tangential Planes are symmetrically placed around this axis and the Polar-curves are Parallel-circles and are sketsing on the solid as the Polar-surface- regular-cone , ***the Porhode*** , around the symmetrical axis , and on the constant system a regular-cone also , ***the Herpolhode*** , around the constant vector $\bar{B}$.

3.1. The Vector-equation of motion .

The vector equation of Rotational-axis-motion is defined by analyzing $\bar{w}$ to $\bar{k}$ direction of Ellipsoid-Inertial-axis , and another one perpendicular to $\bar{k}$, and then $\bar{w} = \bar{i}\, w_1 + \bar{j}\, w_2 + \bar{k} w_3 = \{ \bar{k} [\bar{w}\ \bar{k}] \} + \bar{k}\,\bar{w}$ and since $[\bar{w}\ \bar{k}] = \frac{d\bar{k}}{dt}$ then ,

$$\bar{w} = [\ \bar{k}\ \frac{d\bar{k}}{dt}\ ] + \bar{k}w_3 \qquad \text{……….…… (25a)}$$

Analyzing Momentum-vector $\bar{B}$ as above to $\bar{k}$ direction then exists ,

$$\bar{B} = (\bar{\imath}\, w_1 + \bar{\jmath}\, w_2)\, J_1 + \bar{k}w_3\, J_3 = [\ \bar{k}\ \frac{d\bar{k}}{dt}\ ] . J_1 + \bar{k}w_3 . J_3 \qquad \text{………… (25b)}$$

Moment of external forces Q , to ,O , is equal to → [$\bar{s}\,\bar{Q}$] = - sQ [$\bar{k}\,\bar{k}$`] and Momentum becomes

$$J_1\ \{\ \bar{k}\ \frac{d^2\bar{k}}{dt^2}\ \}\ +\ J_3 w_3\ \frac{d\bar{k}}{dt} + sQ\,[\ \bar{k}\,\bar{k}`\ ] = 0 \qquad \text{…………….…. (25)}$$

Angular velocity w_3 was shown constant .

Equation (25) defines Unit-vector , $\bar{k}$, of Ellipsoid-rotating-axis , related to time and after the solution , equations (25a) and (25b) define Angular velocity , $\bar{w}$, and Angular-momentum-vector $\bar{B}$ respectively .

3.2. The Intrinsic rotation , Precession , and Nutatio

General :

Since Earth is as Sphere , names of longitude and latitude , *which are angles* , are referred to a Right Ascension and Declination in a spherical Polar coordinate system . What is seen from Earth , is the celestial equator on which the Ecliptic , *the apparent path of the Sun through the year* , where the Sun moves into Northern hemisphere and which is called the , *Vernal equinox* , and the analogous motion of the Sun to Southern-hemisphere and called the , *Equinox* .

Sun`s apparent motion is not completely regular and also, both celestial-equator and Ecliptic , are moving with respect to the stars . By far , Precession of the equinoxes is the largest effect , where Earth`s rotation axis sweeps out a cone centered on the Ecliptic pole , completing one revolution in about 26000 years and is called the , *luni-solar precession* . The cause of the motion is the torque exerted on the distorted equatorial bulge of the spinning Earth by the Sun and the Moon.

At the equinoxes equatorial bulge and torque shrink to zero and it is the smaller-faster effect and is called Nutation . Note that Precession and Nutation are simply different frequency components of the same Physical effect .

The orbit of the Earth-Moon system is not fixed in orientation because of the attraction of the Planets , and this slow secular rotation of the Ecliptic about a slowly moving diameter , is the Planetary Precession .

From Classical Mechanics :

The Angular-Kinetic-Energy $\bar{B}$, *Angular momentum vector* , is conserved , so $\frac{d\bar{B}}{dt} = 0$ and since may be expressed in terms of the moment of Inertia Tensor , J , and the Angular velocity vector , $\bar{w}$, so then according to (21a-21b) the ellipsoid of Angular velocity vector is $J_1 w_1^2 + J_2 w_2^2 + J_3 w_3^2 = 2L$ where Kinetic-Energy $L = \frac{\bar{w}^2 J_n}{2}$, and , w , is on the surface of the Inertial Ellipsoid. The tangent

Plane-normal at $\bar{w}$, is $\frac{\bar{w} J_n}{2} = \nabla \frac{1}{2}[\ J_1 w_1^2 + J_2 w_2^2 + J_3 w_3^2\ ] = [\, J_1 w_1 + J_2 w_2 + J_3 w_3\,] = \bar{B}$

i.e. the attitude of the Tangent-Plane is constant and at a distance $\frac{\bar{w}.\bar{B}}{|\bar{B}|} = \frac{2L}{|\bar{B}|}$ which is also constant ,

or from $\bar{B}\ \bar{w} = L = \frac{1}{2}\ J_1 w_1^2 + \frac{1}{2}\ J_2 w_2^2 + \frac{1}{2}\ J_3 w_3^2$, where w_1 , w_2 , w_3 ,are the components of vector $\bar{w}$, along the Principal axes , and J_1 , J_2 , J_3 , are the Principal-moments of Inertia **.** Fig.18 Thus , the conservation of Kinetic-Energy ,L, imposes a constraint on the three-dimensional Angular velocity vector , $\bar{w}$, and an Ellipsoid in the Principal axis frame , *the Inertia Ellipsoid ,* J .

The Ellipsoid axes values are the half of the Principal-moments of Inertia , and the Path traced-out on this Ellipsoid by the Angular velocity vector , $\bar{w}$, is called , *Polhode* .

From above , the Tangent-Plane is Fixed and so , The Energy - Ellipsoid rolls without slipping on this Constant Plane . On Fixed-Constant-Plane is traced the Herpolhode path , while on the Energy-Ellipsoid is traced the Polhode path . i.e. On any Angular-velocity-vector , $\bar{w}$ ***, corresponds an Angular-momentum-vector ,*** $\bar{B}$ ***, to the common point O, of rotation .***

Vector , $\bar{B}$ ***, is perpendicular to the Tangential to ,*** $\bar{w}$ ***, nib Angular-velocity-Ellipsoid , while ,*** $\bar{w}$ ***, is perpendicular to the Tangential to ,*** $\bar{B}$ ***, nib Energy-Momentum-Ellipsoid .*** Figure.14.

According to Euler`s equations (20) , in the Principal axis frame , Angular-momentum-vector (which is rotating in the absolute space) is not conserved even in the absence of applied torques , but varies as in (20). However , in the absence of applied torques , (4a) , magnitude , $\bar{B}$, of the Angular momentum $B^2 = B_1^2 + B_2^2 + B_3^2$ and Kinetic Energy , L , are both conserved and as in (13a) for Angular-velocity-momentum-Ellipsoid $L = \frac{B_1^2}{2J_1} + \frac{B_2^2}{2J_2} + \frac{B_3^2}{2J_3}$, where B_1 , B_2 , B_3 are the

components of the Angular-momentum-vector along the Principal axes , and the J_1 , J_2 , J_3 are the Principal moments of Inertia . These conservation laws are equivalent to two constraints to the three dimensional Angular-momentum-vector ,

$\overline{L}$, The Kinetic energy constrains of , L to lie on an Ellipsoid , whereas the angular momentum constraint , constrains ,L , to lie on a Sphere .
These two surfaces intersect in taco-shaped curves that define the possible solutions for , L .
This is a construction method based on Angular-momentum-vector , $\overline{L}$, rather than that of Poinsot`s which is based on Angular-velocity-vector , $\overline{w}$,

In case of an Axisymmetric Rotating Body , with Angular-velocity ,w, ***the moment of Inertia , J , about two of the Principal axis ,*** x - y ***, are equal , then The Angular – velocity - vector ,*** $\overline{w}$ **,** describes the Ellipsoid of Angular Velocity and its nib describes a Cone of which Plane-Base is Fixed ***, and Simultaneously ,*** **The** ***Angular-Momentum*** , $\overline{B}$, describes the Ellipsoid of *Angular-Momentum and its nib describes* a Cone also of which Plane-Base is also Fixed **.**
The Nib of Angular – velocity - vector , $\overline{w}$ ***, describes on the Tangential-Plane of the Angular-Momentum-Ellipsoid ,*** *the Herpolhode* ***, while , The Nib of Angular-Momentum describes on the Tangential-Plane of the Angular-Velocity-Ellipsoid ,*** *the Polhode* **.**

The Fixed-Tangential-Planes of , $\overline{w}$ *, and ,* $\overline{B}$ ***, are alternately Perpendicular to ,*** $\overline{B}$ ***, and ,*** $\overline{w}$ ***, central axes of rotation and thus form the Material-Point-energy-monad .***
Vector , $\overline{B}$ ***, is perpendicular to the Tangential to ,*** $\overline{w}$ ***, nib Angular –velocity -Ellipsoid , while ,*** $\overline{w}$ ***, is perpendicular to the Tangential to ,*** $\overline{B}$ ***, nib Energy-Momentum-Ellipsoid.***
All above happen in Material-Point , where the , ⊕ ***, constituent is Eternally self-rolling on the***
, ⊖ ***, constituent ,*** with Angular-Velocity , **w ,** becoming from constant constituents Glue-Bond Pressure , **σ** , ***in Infinite Spherical traces , either at Great-circles or Small-circles , or any other close Spherical-curve , and by applying all laws of Mechanics into this Energy-Chaos , is thus created the First-Discrete-Energy-monad which is the Material -Point .***

Equations (24)-(25) , show immediately if the motion is Possible , and under which circumstances. To examine the Possibility for Ellipsoid-Symmetrical-axis ,OS , to perform rotation as Regular-Cone around the vertical axis $\overline{k}`$, **i.e**. if it is possible , under the presupposition , ***ϑ = constant*** , to solve above referred equations of motion . Because w_3 = constant , from (24d)

Second-equation implies $\frac{d\varphi}{dt}$ = constant , while

First-equation implies $\frac{d\psi}{dt}$ = constant

Meaning that , according to equation (19) , $\overline{w} = \overline{k}` \frac{d\varphi}{dt} + \overline{k}\ \frac{d\psi}{dt}$ ………… (26a)
and which is considered as the rotation of Material-point around the Ellipsoid-Symmetrical-axis ,OS , with constant Angular-velocity u= $\frac{d\psi}{dt}$,and simultaneously rotated through the vertical $\overline{k}`$ axis with the same constant angular velocity $\frac{d\varphi}{dt}$ =u . Since Angular-velocity-vector $\overline{w}$, is constant then its algebraic-figure is also constant and lies in , $\overline{k}\,\overline{k}`$, Plane .

Because [$\overline{k}\,\overline{k}`$] = - $\overline{s}_o$ sin ϑ , and $\frac{d\overline{k}}{dt} = [\overline{w}\ \overline{k}]$ and from (26a)

$\frac{d\overline{k}}{dt} = \overline{s_o}\ \sin\vartheta$, $\{\ \overline{k}\ \frac{d\overline{k}}{dt^2}\ \} = [\ \overline{k}\ \overline{s_o} . u. \sin\vartheta\ ] = \overline{t_o} . u. \sin\vartheta$ and

$\frac{d}{dt}\ [\ \overline{k}\ \frac{d\overline{k}}{dt}\ ] = [\ \overline{k}\ \frac{d^2\,\overline{k}}{dt^2}\ ] = \frac{d\overline{t_o}}{dt} . u. \sin\vartheta$ and because

$\frac{d\overline{t_o}}{dt}$ = the nib of velocity-vector $\overline{t}_o$ = - $\overline{s}_o$. u. cos ϑ then , $[\overline{k}\ \frac{d^2\,\overline{k}}{dt^2}] = -\ \overline{s_o} . u^2 . \sin\vartheta . \cos\vartheta$ Introducing above in (25) and , division by sin ϑ , and when 0 < ϑ > 0 , and , π < ϑ > π , then becomes relation $J_1 . u^2 \cos\vartheta - J_3 w_3 .u + s .Q = 0$ ……………. (26) which is the Necessary-Proposition , *Precession* , for the motion of the Material-Point [⊖↔⊕]. Second degree Equation (26) gives real roots for velocity , **u** , only for negative cos ϑ , i.e. the center of mass , S , is below rotational point O. If cos ϑ > 0 then $w_3{}^2\ >= \frac{4\,J_1 sQ. \cos\vartheta.}{J_s{}^2}$ ……. (27)

and by solving (27) then Angular-velocity u= $|\overline{w}| = \frac{d\varphi}{dt} = \frac{J_3 . w_3}{2\,J_1 . \cos\vartheta} \pm \sqrt[2]{-\frac{sQ}{J_1 . \cos\vartheta} + (\frac{J_3 . w_3}{2\,J_1 . \cos\vartheta})^2}$ … (27a)
Since equation (27a) , is of 2nd degree and has two solutions ,a, and ,b.
a.. Algebraic-magnitude-figure w_3 is such that , the within square root is zero or near zero . From figure this happens when figure w_3 coincides with that of Angular velocity vector , $|\overline{w}|$, and from $\frac{d\psi}{dt}$, where then its direction coincides with the Ellipsoid-Symmetrical-axis ,OS. The second term being in square is strengthening J_1 moment of inertia directing

the axis to J_3 direction. Material point occupies a large moment of Inertia by rotating about the Ellipsoid-Symmetrical-axis through constant point O, on Ellipsoid-axis , of the truncated Cone , i.e.
The Inner-Rotation of Material-point , happens through the algebraic and constant Angular-velocity-vector , $|\overline{w}|$, { ***on the Ellipsoid-Symmetrical-axis } , where the Surface of the Truncated-Polar-Cone is Rolling ,*** sweeps out **, *on the Unmovable-Polhode-Cone with a very large figure ,*** *the absolute value* ***, of Angular velocity vector ,*** $|\overline{w}|$ **.**
In Material-point where External forces are equal to zero (Q=0) , the above equation becomes ,

Angular-velocity-figure , u= $$|\overline{w}| = \frac{d\varphi}{dt} = \frac{J_3 . w_3}{2\,J_1 . \cos\vartheta} \pm \sqrt[2]{\left(\frac{J_3 . w_3}{2\,J_1 . \cos\vartheta}\right)^2} = \frac{J_3 . w_3}{2\,J_1 . \cos\vartheta} + \frac{J_3 . w_3}{2\,J_1 . \cos\vartheta} = \frac{J_3 . w_3}{J_1 . \cos\vartheta}$$ and

when $J_1 = J_3$ then , u = $\frac{w_3}{\cos\vartheta} = w_3 .\ \sec\vartheta$, meaning that the constancy of ,u, becomes from Geometry of the rotating energy only as this happens to the constancy of velocity , **c** , in cave , **r** . This is the analogous which happens to Lorentz factor $\gamma \equiv$ ***sec.φ , and then*** u = w_3. sec $\vartheta = \gamma .\ w_3$

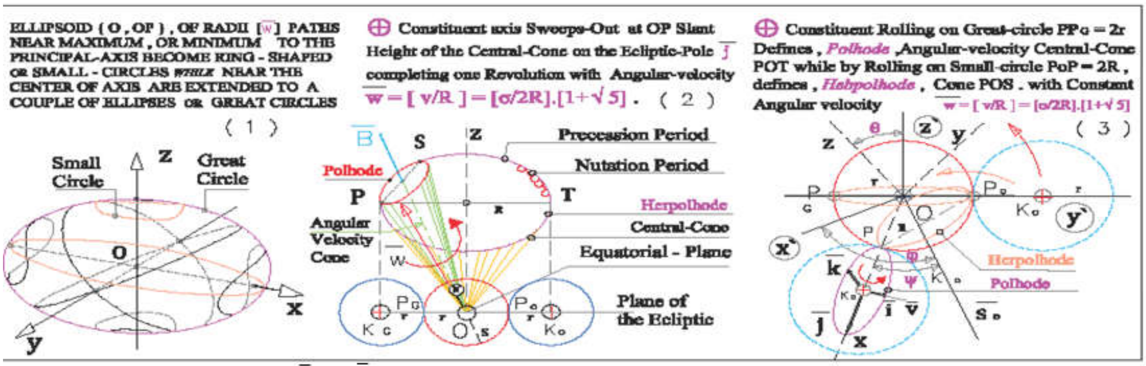
b.. Algebraic-magnitude-figure ,u, is equal to w_3 and $\frac{d\psi}{dt}$ and to ,w, also .
In this case , the direction of the Angular-velocity-vector $\overline{\mathbf{w}}$ is diverging the $\overline{\mathbf{k}}$ axis of the Central-Truncated-Ellipsoid and Angular-velocity ,**u**, occupies , low and high values.
This Phenomenon happens in Astronomy where Equator-points , i.e. on Celestial-Sphere traces of sections ($\overline{s}_o$) of the Planes of ecliptic ($\overline{k}$`) and Ecuador ($\overline{k}$) , are slowly moving on Zodiac circle with an approximate Period of 21000 years .

Due to Earth-Geoid , Precession of Equator-points is proved to become from rotation of the Earth Polar-axis perpendicular to Ecliptic .
c.. The algebraic figure within the square root is zero or near zero .
In case , ⊕ , constituent is rolling on great circles on ⊖ constituent then , all moments of Inertia of Material point are equal and $J_1 = J_2 = J_3$ and angular velocities of mass center are also equal therefore $w_1 = w_2 = w_3$ and sec $\vartheta = 1 / \cos\vartheta$, therefore (27a) becomes ,

Angular-velocity u = $$\frac{d\varphi}{dt} = \frac{J_3 . w_3}{2\,J_1 . \cos\vartheta} = \frac{w}{2\cos\vartheta} = \frac{w}{2}(\sec\vartheta) = \frac{w}{2\sqrt{1-(\frac{v}{c})^2}} = \frac{w.c}{2\sqrt{c^2-v^2}} =$$
$$\frac{wr.c}{2\sqrt{c^2r^2-2\sigma^2.[3+\sqrt{5}]}} = \frac{v.c}{4\sqrt{c^2r^2-2\sigma^2.[3+\sqrt{5}]}} = \left[\frac{\mathbf{c.\sigma}}{\mathbf{4r}}\right].\frac{[\mathbf{1+\sqrt{5}}]}{\sqrt{\mathbf{c^2r^2-2\sigma^2}.[3+\sqrt{5}]}} = \mathbf{u} \quad .....(c)$$
Equation (c) , defines the Angular-velocity-figure ,**u,** related to constant velocity ,**c ,** and Stress , **σ** .

i.e. Constant Angular-velocity-value ,u **,** ***is such because of the two constants* , c ,** ***and Glue-Bond* , σ ,** *or* , ***The Energy of opposite* , ⊖ , ⊕ , *from Chaos*** { r ≡ 0 } , ***is transformed as Discrete - Monad*** | ⊖ ⊕ | ***in the Self -Rotated Material-Point*** {| ⊖ ↔⊕ |} , ***due to the Glue-Bond Stress* , σ .** [58] **d..** The Eternal Precession .
Torque , T , is the Twisting - Force that tends to cause rotation **{** T = Q.r sin ψ where Q is the force and ,ψ, the angle between radius ,r, and force Q **}** and the Angular-momentum-vector , L = J .w . ***Material - point is a System which has an Inner – Rotation - constrained*** , *Due to the velocity vector ,* $\overline{v} = \frac{d\psi}{dt}$ *becoming from Stress ,* ***σ*** *, which is the Force-applied on lever - arm , r* ***, in space , on where External Forces and Moments are not existing*** .
The inner forces of this system , are the two equilibrium Centripetal and Centrifugal Forces due to the Eternal , ± σ **,** ***Stresses of Opposites*** .

Equation (26) is the Necessary-Proposition , *Precession* , for the motion of the Material-Point In case that is not holding then Precession near the axis of Ellipsoid is continuously existing . Considering the Material-Point rotating with a great speed $\frac{d\psi}{dt}$, around a fix point O, and near the symmetrical axis of the Central Ellipsoid , and lying on this axis also then , because of the great velocity exists a great Angular velocity and therefore a strong Rotational-Momentum , so that change is very small as , $d\overline{B} = \overline{M}dt = [s\,\overline{Q}]\,dt = -\,s\,Q\,[\overline{k}\,\overline{k}`]\,dt$, and for a short of time it is very small to real Momentum , as (|B|>>|s Q|) . The same also happens to Kinetic energy for a small displacement on ⊖, constituent . In figure.17-1 and F.18-3 , the ⊕ constituent executes different rings near maxima or minima. Around the fixed Vector of Momentum , $\overline{B}$,is moving the Angular-velocity-vector , and Inertial Ellipsoid is Rolling on Poinsot`s constant Plane , following a Circular Polar curve and , the animated rolling on the fixed Central-Pole-Cone , drawing the symmetrical axis , Oz ,to Precession . Because Vectors , $\overline{\mathbf{B}}$ and $\overline{\mathbf{w}}$ **,** are very near each other and both to $\overline{\mathbf{k}}$ axis , Polar curves are very narrow rings as in figure , the Animated on the Ellipsoid of that of Angular-velocity while the Un-movable on Poinsot`s Plane and around Vector , $\overline{B}$. During motion , Momentum , $\overline{B}$,is altered because of the different moment of Inertia due to the



Position of Vectors , $\bar{B}$, and k` axis .

The above is happening because of the equivalence of Kinetic-Energy and the Rotating-Energy , as

L = (1/2). $\Sigma m_i.\bar{v}_i^2$ = (1/2).[Σm_i].$\bar{v}_s^2$=(1/2). $\Sigma m_i.[\bar{w}.\bar{r}_i]^2=(1/2).(\Sigma m_i.\bar{r}_i^2).\bar{w}^2 = \frac{1}{2}J_a w^2$ i.e.

Figure.19. In (1) , are shown different Paths near maximum or minimum Principal axis from where become-ring shaped or Small-circles while near the center of axis extend to a couple of ellipses or Great-circles .For a constant Angular-velocity ,**w**, and by changing torsional momentum ,**B**, only then Infinite curves are possible for the motion .

Rotational Kinetic Energy L = $\frac{1}{2}J_a w^2$ = (1/2).[Σm_i].$\bar{v}_s^2$ = L = *Kinetic Energy , is dependent on the moment of Inertia which is related to the Area of the curve* , so Rotational-Momentum-vector $\bar{\mathbf{B}}$, is slowly rotated on Central axis with angular velocity , **u** , which is $\frac{d\bar{B}}{dt} = -$ sQ [$\bar{k}\,\bar{k}$`] . Integrating

(Angular velocity , u =) becomes → $\frac{sQ}{B} = \frac{sQ}{J_3 w}$ and the Period of rotation T = $\frac{2\pi}{u} = \frac{2\pi\ J_3.w}{sQ}$.

Since also frequency f = 1 / T , then f = $\frac{sQ}{2\pi.J_2.w}$.

In (2) , ⊕ constituent is rolling on Great-circle P_GP_o = 2r Sweeps-out , at OP slant height of the Central-Cone of Angular velocity on the Ecliptic-Pole ,$O\bar{J}$, with angular velocity , w = $\frac{v}{r}$. Angular-velocity-Ellipsoid describes Central-Cone POT with , *Herpolhode* , on Fixed Base-circle PT while Momentum-Ellipsoid describes Cone POS with , *Polhode* , on Fixed Base-circle PS. The tangential-Plane of Angular-velocity-Ellipsoid at , P, is perpendicular to Angular-momentum-Ellipsoid axis $\bar{B}$, while the tangential-Plane of $\bar{B}$ vector is perpendicular to Angular-velocity-Ellipsoid $\bar{w}$ vector .

In (3) , ⊕ constituent is rolling on Small-circle PP_o= 2R , Sweeps-out , at OP slant height of the Central-Cone of Angular velocity , *Polhode* , on the Ecliptic-Pole,$\bar{J}$, with the angular velocity , w = $\frac{v}{r} < \frac{v}{R}$ such that , the Rotational-momentum $\bar{B}$ becomes ,

$$\bar{B} = \bar{J}\ J_2 . \frac{v}{R} = \frac{\pi r^4}{4} . \frac{\sigma}{2r} [1+ \sqrt{5}\,]= \frac{\pi r^3 \sigma}{8} [1+ \sqrt{5}] .$$

Because vertical force ,Q, cannot be zero and Angular velocity , u = $\frac{sQ}{B} = \frac{sQ}{J_3 w}$ = constant and Period of rotation , T = $\frac{2\pi\ J_3.w}{sQ}$ changes , and this from , J_3 , by the area of the New Great- circle (πR^2) instead of Initial (πr^2), becomes Q = $\frac{u\bar{B}}{s}$

3.3. ***Application to Material-Points*** [⊕↔⊖] of Figure.19

Since the constituents of Material Point are without mass , the Rotational Momentum becomes from Angular-velocity ,w, and so equation of *Rotational Kinetic Energy* L = $\frac{1}{2}J_a w^2$, is related to moment of Inertia J_a , *therefore* any change to the Great-circle-area of Ellipsoid , changes Kinetic-Energy , so , On ⊕, constituent Inertia J_a becomes from the Inertia of Area , of the Surface traced on the Great circle of the ⊖ constituent and is $J_3 = J_r = \frac{\pi r^4}{4}$ for circle-radius , r , and of Area = πr^2 , while on Small-circle of radius R is $J_3 = J_R = \frac{\pi R^4}{4}$ and of Area = πR^2 .

The change in moment of Inertia is $J_D = \frac{\pi}{2}(r^4 - R^4)$ and becomes of the different trace of motion .

The ⊕, constituent is rolling on ⊖ constituent with the constant velocity $\bar{v} = \frac{\sigma}{2} [1+\sqrt{5}]$ because of the Constant Stress ,σ. Circular motion happens with parallel circles perpendicular to ,z , axis .

Case a.. ⊕, constituent is ***Not-Rolling around ,z, axis .***

The Algebraic-figure of Momentum to center of mass is $J_2. w_2$, where J_2 is the moment of Inertia to vertical axis y , and w_2 is the Angular-velocity-meter and equal to $\frac{v}{R}$, where R is the Curve-Radius of Curvature .

The direction of the Momentum is on , y , axis and that of Angular-velocity-vector is $\bar{i}\, w_2$ and issues , $\bar{B} = \bar{i}\, J_2\, w_2 = \bar{i}\, J_2 . \frac{v}{R}$

Because the first derivative of Momentum is zero therefore magnitudes , v , R , J_2 are unaltered and so is not needed any other force to act on , z , axis , *except that of Centripetal* , to follow curve.

Case b.. ⊕, constituent ***Is-Rolling around ,z, axis on Small-circles*** .

Because of the constant Stress , σ , and Angular velocity w_2 , Momentum $\bar{B}$ is containing another term on $\bar{k}$ axis , $\bar{k}\, J_3\, w_3$ with much greater Algebraic-figure , and this because during rolling on semicircle , another curve R < r on ⊖ constituent executes greater number of turns about its axis , x

and $J_3 = \frac{\pi R^4}{4} < \frac{\pi r^4}{4}$ and the difference moment of Inertia is $J_D = \frac{\pi}{2}(r^4 - R^4)$.

Angular velocity w_3 becomes greater to w_2 as it is , $J_R > J_2$. The first derivative of this term under the restriction w_2= constant becomes $\frac{d\bar{B}}{dt} = J_3\, w_3 \frac{d\bar{k}}{dt}$, where $\frac{d\bar{k}}{dt} = \bar{i}\, w_2 = \bar{i}\, \frac{v}{r}$, so for the continuity of motion is needed a couple of forces at the end points of axis such that moment is

$\bar{M} = \bar{i}\, J_D\, w_3 w_2 = \bar{i}\, J_D\, w_3 . \frac{v}{r}$. Reactions are created at the ends of , z , axis and for 2R distance

then Reaction is $F = \frac{J_D\, w_3 w_2}{2R} = \frac{J_D\, w_3\, v}{2R^2} = \frac{J_D\, v^2}{2R^2\, r} = \frac{\pi(r^4 - R^4)\, \sigma^2 (3+\sqrt{5})}{2R^2 r. r^2} = \frac{\pi\sigma^2 (r^4 - R^4).(3+\sqrt{5})}{2\, R^2\, r^3}$,

which is the Gyrostatic reaction of motion .

The vertical force $Q = \frac{u\bar{B}}{s} = \frac{u\bar{B}}{R} = \frac{\frac{\sigma}{r}[1+\sqrt{5}] . \frac{\pi r^3 \sigma}{8}}{R} [1+\sqrt{5}] = \frac{[\pi\, \sigma^2\, (3+\sqrt{5})]}{8 . R}$ and for rotation the needed moment is , $M = Q(r^2 - R^2) = \frac{[\pi\, \sigma^2\, (r^2 - R^2)\, (3+\sqrt{5})]}{8}$ with Reactions on perpendicular to , y , axis .

For Planck`s length and ratio $k = \frac{R}{r} = 0,5$, the above become in Algebraic figures ,

$$F = \frac{\pi\sigma^2 (r^4 - R^4).(3+\sqrt{5})}{2\, R^2\, r^3} = \frac{\pi\sigma^2 (1 - k^4).(3+\sqrt{5})}{2\, k^2\, r} = \frac{\pi.17{,}04.10^{-54}.0{,}75\, .5{,}236.}{2.0{,}25\, .4{,}453.10^{-35}} = 9{,}442\, .\, 10^{-18}\ N$$

$$Q = \frac{[\pi\, \sigma^2\, (3+\sqrt{5})]}{8 .\, R} = \frac{[\pi\, \sigma^2\, (3+\sqrt{5})]}{8 .\, k r} = \frac{\pi.17{,}04.10^{-54}}{8.0{,}5\, .4{,}453.10^{-35}} = 3\, {,}000\, .\, 10^{-19}\ N$$

$$M = \frac{[\pi\, \sigma^2\, (r^2 - R^2)\, (3+\sqrt{5})]}{8} = \frac{[\pi\, \sigma^2 r^2 (1 - k^2)\, (3+\sqrt{5})]}{8} = \frac{\pi.17{,}04.10^{-54} 0{,}75.19{,}829.10^{-70} 5{,}236.}{8} = 1{,}327.10^{-121}\ KN.m$$

Remarks :

Forces , F , Q , applied on lever-arms , r , and , R , *which are both in the* , Material-point-System , are differing on the Moment of Inertia J_r and J_R values , of the sketching circles of rotation .

The Material-point *is the discrete continuity* ***Content*** ≡ |{⊕+⊖}|= ***The Quantum*** through mould of Space –Anti-space in itself , *which is the material dipole in inner monad Structure and is Identical with the Electromagnetic cycloidal field of Energy monads .This is ,*the Energy-distance ≡ **The Form** *, and consists the deep concept of Material-geometry ,* ***i.e. Material-Point becomes as , DISCRETE - FORM , from Euclidean-Point which is the CHAOS , by the Eternally-Moving-Content → the in Mode Content of existence , which is the Energy-Quanta in Mechanics. In Article is clarified the*** **How** ***, the*** **When** ***and the*** **Why** ***CHAOS becomes DISCRETE and thus Joining Euclidean-Geometry – Mechanics – Physics in One Unity and with the same Universal Laws , from Zero ≡ Non-existence ≡ Chaos , to Discrete , Microcosms to Macrocosms .***

In Primary*-material-point ,* ***Form*** (r) ***and Content,*** [⊕↔⊖] *, is constant while in all others issues the law of transformation of* ***Quantity into Quality*** *, and this is extended from the smaller particle to the largest phenomena , i.e. in all levels of the Energy-space universe .*

Form (r) ,**of *Material point*** **AB** ,{ *the two Spherical constituents* ,[⊖≡ K_o,K_oB]–[⊕≡K,KB]} *of Space point* , A (⊕) *and Anti-point* , B (⊖) ***and created on STPL Mould*** , *by the rotating velocity vector* v_A = w.r , ***is thus forming*** *the material angle* , $\vartheta = \vartheta_A . t = (\frac{v_A}{\sqrt{c^2 - r^2}})$. t , *which in turn* { **r** and **ϑ** } *create the Envelope of Harmonic-Vibration-curves on this , r , ϑ rotating STPL line* . In [58] ,

*The transformation of **Quantity into Quality** can be seen in many levels as , the velocity* v_A *of any point* A , *in Primary-Dipole* AB *creating Infinite Free Harmonic Vibrations on* AB *monads , following Euler-Savary mechanism where Rolling motion is transformed to Vibration curves , and on different waves which properties is determined by the number of oscillations per second , i.e. the **frequency related to vibrations** is the quantitative change giving rise to different kinds of the wave-signals. Increasing the rate of vibrations turns the colours from Red indicating low frequency , to Orange - Yellow , to Violet , to the invisible Ultra-violet and ,* X-*rays , and to gamma rays .*

Reversing the process at the lower end , we go from infrared and heat rays to radio-waves , as in [39]. *Also the changing of Temperature offers no resistance to electric-currents and for Helium which is the only substance which cannot be frozen because is in Primary-Form and exists the Critical – Energy -Quantity* CEQ *as before . The difference between Organic and Inorganic Chemistry is only relative , i.e. the different collection of atoms and the DNA structure.*

The elementary particles which make up the atoms interact constantly by passing into each other while at every moment are both themselves and something else (are a different entity which in turn determines the behavior of its component parts) while the Union of atoms into molecules follows chemical formulas, the atoms themselves had remained unchanged with only a purely quantitative relationship , in contradiction to Molecule which cannot be reduced to its component parts without losing its identity . The Principle of the < Whole being equal to the parts > issues for all compositions either in Form or in h , as happens to the square root of a number which can be either positive or negative .

It was referred that the Zero equality in Content , **(**$P_A + P_B$**)**=0 *is the Critical-Energy-Quantity* {CEQ} for *any transition in Quality , is a kind of Catalyst which is not changing the composition of Primary Segment ,* the unity of opposites *and also the Work ≡ Energy ≡ Heat ≡ Pressure ≡ velocity ≡ motion involved in all levels , and generally on Material-Points , in Material-Geometry . Beyond a certain Critical-Energy-Quantity the Bonds ≡ Content are broken and then a qualitative leap occurs .*

AS Zero(0) *is the Border-line between all Positive* (+) *and Negative magnitudes* (-) *and stands in a relation of infinity to every other number and represents a real magnitude ,THUS* {CEQ} *which is zero in Material-point , is identified with that Zero of Euclidean-geometry.*

Quality in a particle is ± Spin dependent on its direction , giving the Outer Electromagnetic-Wave of moving and the Inner Electromagnetic-Field of monads . This Inner unity of opposite is , in nature , the velocity ≡ motor-force of all motion , starts to recover → gathering strength as Spin which in turn to Outer-Spin and to the Electromagnetic-Wave . [40]

All above occur either by Rolling of Space A ≡ (⊕) *on Anti-space* B ≡ (⊖) sphere , joint by ,σ, Stress , *on STPL Mould* , or *by Rolling of Space* A ≡ (⊕) *on Anti-space* B ≡ (⊖) Evolute-Cycloid , *joint by the* ,$\bar{\nabla}$, energy , *on STPL Mould-length which is the* → Isochronous curvature radius *from the Cycloid , Evolute-cycloid Rolling points* . All above is a Slit for Future-Technologies .

4.. Epilogues .

The origin of Space [S] *becomes , through the Principle of Virtual Displacements* $W = \int_A^B \boldsymbol{P}.\boldsymbol{ds} = 0$ *, from Primary Point* **A ,** *which is the Space , to point* **B** *which is the Anti-space as the Inner distance of Space and Anti-Space in all Layers becoming as shown from STPL Mechanism .*

The origin of Energy *becomes , through the same Principle because are co-related and is the Work executed by the displacement ,* ds , *which is conserved and never vanishes .*

This means that Universe is Energy-Space and nothing else , which follows the Glue-Bond - Principle in all Positions and Layers starting from

The First Eternal < Self – Moving – Energy - Dipole > ≡ The Quantum , of this cosmos*.*

The Torsional oscillation *of Caves (cleft , slit)* ***w*** *, is transformed as inner Wave-frequencies which in turn , to monads and moving Particles transforming Inward-Spin to the Outward-Spin and motion . All above are produced in and from STPL .* ***Energy produced*** *by Reference System*
{D_A- P_A}≡[R] (x',y',z', t') *moves with velocity* ,$\bar{\nabla}$, *parallel to* , x-x', *axis with respect to the fixed and Absolute System* {$\mathbf{D_A}$- O} ≡ [S](x,y,z, t) and is conserved .

Energy of the whole universe *is defined as a whole , all at once , and not , the Energy of different pieces.*

It was referred that Energy in Gravitational – Field is Torsional and Negative and always attractive . [27]

In General-Relativity *is referred that Space time is giving energy to matter or absorbed it from matter ,and thus the Total energy is not conserved . Here are not clarified the three Basic Quantities ,* ***Energy , Matter and Time***

The Material-point *is the discrete continuity* ***Content*** ≡ |{⊕+⊖}|= ***The Quantum*** through mould of Space –Anti-space in itself , *which is the material dipole in inner monad Structure and is Identical with the Electromagnetic cycloidal field of Energy monads .This is ,*the Energy-distance ≡ **The Form** *, and consists the deep concept of Material-geometry , **i.e. Material-Point becomes as , DISCRETE - FORM , from Euclidean-Point which is the CHAOS , by the Eternally-Moving-Content → the in Mode Content of existence , which is the Energy-Quanta in Mechanics. In Article is clarified the*** **How** ***, the*** **When** ***and the*** **Why** ***CHAOS becomes DISCRETE and thus Joining Euclidean-Geometry – Mechanics – Physics in One Unity and with the same Universal Laws , from Zero ≡ Non-existence ≡ Chaos , to Discrete , Microcosms to Macrocosms .***
In Primary*-material-point ,* ***Form*** (r) ***and Content****,* [⊕↔⊖] *, is constant while in all others issues the law of transformation of* ***Quantity into Quality*** *, and this is extended from the smaller particle to the largest phenomena , i.e. in all levels of the Energy-space universe .*
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