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THE APPLICATION OF EVOLUTIONARY GAME IN RESOURCE ALLOCATION

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Abstract:-
Resource allocation could not arrive at Nash equilibrium directly as that under completed rationality, due to bounded rationality of users. A resource allocation strategy based on evolutionary game is proposed to investigate the evolutionary process of user colony from the dynamic viewpoint. Using the method of replicated dynamics, an evolutionary stable strategy is produced to allocate resource. In particular, the evolutionary stable point, evaluation function characteristics, and replicated dynamic diagrams are discussed under different conditions. The results show that using evolutionary game approach, users could study and adjust strategy constantly through repeated games to achieve evolutionary stable equilibrium, which leads to an optimal allocation of resource.

Keywords:- *Resource allocation; Nash equilibrium; evolutionary game*

1. INTRODUCTION

Cloud computing [1,2] is the delivery of computing services—including servers, storage, databases, networking, software, analytics, and intelligence—over the Internet to offer faster innovation, flexible resources, and economies of scale. In Cloud system, resource management is very similar to the one in social economic activity since they are all based on distributed autonomy and related to game problems with conflicting objectives among interacting decision makers. Nowadays, many researches have explored the idea of game theory [3] to resource management. These researches formulate the resource configuration as a game problem and obtain the optimal allocation through searching Nash equilibrium solution.

According to the type of game player, those researches can be classified into two main categories. One is focuses on the game between resource and user. These methods [4-6] cannot match well with a practical computing platform in which the users interacted with each other. Another is focuses on the game among the users. But, these methods [7-9] are all based on completed rationality hypothesis that assume players aim to maximize their own benefits all the time and have abilities of perfect judgment and prediction. In fact, users are bounded rational because they often make mistakes. That means the strategy equilibrium between users can only be arrived by dynamic adjustment not one-off selection.

This paper proposed a bid strategy of bounded rational users in resource allocation using evolutionary game theory [10, 11]. The rational degree of users can be improved by introducing dynamic studying process. Through repeated games, users can adjust their strategies to arrive at the final bid equilibrium that leads to the optimal allocation of resource.

2. Resource Allocation Problem

The system has N users competing for a computing resource with fixed finite capacity. These users are given a job of completing a sequence of tasks of different types by purchasing resource. The resource is allocated using the proportional resource sharing mechanism, where the percentage of resource share allocated to the user is proportional to the bid value in comparison to other users' bids. Suppose that each user needs to submit a bid b_i to obtain resource capacity. Then, the resource partition allocated to the ith bidding user $w_i(b_i) \in [0,1]$ satisfies

$$w_i(b_i) = b_i \Big/ \sum_{j=1}^{N} b_j \quad (1)$$

Let c_i be the capacity of the resource chosen by the ith user to complete its job, and B be the total bids that the resource receives from all users, i.e. $B = \sum_{i=1}^{N} b_i$. Then the resource capacity allocated to the ith user is

$$r_i = c_i w_i(b_i) = c_i (b_i / B) \quad (2)$$

Even through the resource allocation is accomplished through a bid mechanism, we note that ultimately each user pays the same price per unit resource obtained. The resource allocation can be interpreted as a resource sold at a uniform price where the price is determined by the users. We assume that each user has a valuation $v_i(w_i)$ for receiving an allocation w_i. This valuation may be a characterization of the estimated performance as a function of a given share of the resource. If the valuation of user is continuously differentiable, then the utility of user can be defined as the difference between valuation and cost.

$$U_i(b_i) = v_i(w_i(b_i)) - b_i \quad (3)$$

According to completed rationality assumption, user bids to maximize its utility all the time. Let the derivative of utility function (3) with respect to b_i equal to zero.

$$U_i'(b_i; B) = v_i'(w_i) w_i'(b_i) - 1 = 0 \quad (4)$$

To obtain the optimal user bid b_i which is function of B . A Nash equilibrium solution is a set of user bids $\{b_i^*\}_{i=1}^{N}$ where no user can gain an advantage by unilaterally changing its bid. This Nash equilibrium can be solve by equation (5)

$$B = \sum_{i=1}^{N} b_i(B) \quad (5)$$

However, the above analysis is based on the completed rationality that assume user try to maximize its own utility all the time. It is unpractical if users can not make any mistakes. To solve the problem, an essential way is to base the game analysis on the bounded rationality of users.

3 Evolutionary Game under Bounded Rationality

3.1 Evolutionary Game Model

Evolutionary game theory emphasizes on the dynamic adjustment of system equilibrium. We explore the replicated dynamic mechanism [12] to describe the dynamic process because replicated dynamic mechanism could describe exactly the relationship between the payoff of individual action and the population evolutionary. Replicated dynamic mechanism means the increasing rate of the percentage of players that select certain strategy is equal to the difference between the payoff of player that select this strategy and the average payoff of population. The replicated dynamic equation is

$$dx_i \big/ dt = x_i[f(s_i, x) - f(x, x)] \quad (6)$$

where x_i is the percentage of players that select strategy s_i at time t ; dx_i/dt represents the change of this percentage with time; $f(s_i, x)$ represents the expectation utility of individuals that select pure strategy s_i ; $f(x,x)$ represents the average expectation utility of population, $f(x,x)=\sum_i x_i f(s_i,x)$. The equilibrium point exists when replicated dynamic equation is equal to zero. Hence, evolutionary stable strategy can be derived from replicated dynamic equation. The definition of evolutionary stable strategy is as follows.

Definition 1. Given $x \le x^*$, where $x^* \in (0,1)$, if and only if $f(s, x) > f(s', x)$ is satisfied for any strategy $s' \neq s$,strategy $S \in S$ is an evolutionary stable strategy.

Here, S is a set of mix strategy. In this context, mix strategy means the distributed probability of individual that select different pure strategy at some time in population. Strategy s is said to be evolutionary stable strategy. System arrive at evolutionary stable status when all individual select strategy s .

3.2 Game System of Resource Allocation

Resource allocation can be formulated as a dynamic game problem that study the change of user bid strategy with time. The following are the main characteristics of the game of resource allocation.

① Symmetric population. Although each user should be up against all the other users, strategy selection of user can be assumed to perform between a pair of user.

② Mutation strategy. There are two types of bid strategy, i.e. primary and mutation. The primary strategy is selected by all players at begin.

③ Slow learning. Players try other strategy and find their way to a strategy that work well. So transformation to dominant strategy can only be reached gradually.

④ Utility oriented. During the repeated games, good strategy brings user more utility, whereas bad strategy will be choked back.

For the sake of simplicity in analyzing, the users in game system are abstracted as a population of two bounded rational users. During the process of bid game, each user faces two selections of bid strategy: primary strategy and mutation strategy. The utility level of different bid strategy is calculated by equation (3). Now, we have established a game frame of random partnership expressed by a 2×2 matrix as figure 1. A user is situated on position 1 is the same as that on position 2 because two players are symmetric in both strategy and benefit.

Table 1. Utility matrix of bid strategy selection for user

		user 1 Bid E/k	Bid E
user2	Bid E/k	$v\left(\frac{1}{2}\right)-\frac{E}{k},v\left(\frac{1}{2}\right)-\frac{E}{k}$	$v\left(\frac{k}{k+1}\right)-E,v\left(\frac{1}{k+1}\right)-\frac{E}{k}$
	Bid E	$v\left(\frac{1}{k+1}\right)-\frac{E}{k},v\left(\frac{k}{k+1}\right)-E$	$v\left(\frac{1}{2}\right)-E,v\left(\frac{1}{2}\right)-E$

In Table1, E/k is primary strategy,E is mutation strategy, where Eis budget,kis a constant more than one.kEv−)21(is utility obtained when two users both select primary strategy, Ev−)21(is utility obtained
When two users both select mutation strategy, $v(1(k+1))-E/k$ is utility obtained when user selecting primary strategy encounter user selecting mutation strategy, $v(k/(k+1))-E$ is utility obtained when user selecting mutation strategy encounter user selecting primary strategy.

Population is initialized as players both select primary strategy E/k . If two players select same strategy, they will obtain the same utility. This is because our resource allocation is based on proportional sharing mechanism. If one player select mutation strategy, then the player must occupy the resource partition owned by another player that select primary strategy at more cost. However, under bounded rationality, player is unaware of the better strategy. They cannot help but learn how to play games through trial and error. They experiment with strategies, observe their payoffs, try other strategy and find their way to a strategy that works well. Next section, we will solve the evolutionary stable equilibrium of the game.

3.3 Solution to Evolutionary Stable Equilibrium

If the proportion of users that select primary strategy E/k is x(0□x□1), then the proportion of users that select mutation strategy $E\ k$ is $1-x$. Using the evolutionary game model, we can calculate the expectation utility of selecting different strategies and the average expectation utility of user population as:

$$f(\frac{E}{k},x)=x\cdot\left(v\left(\frac{1}{2}\right)-\frac{E}{k}\right)+(1-x)\cdot\left(v\left(\frac{k}{k+1}\right)-E\right)$$

$$f(E,1-x)=x\cdot\left(v\left(\frac{1}{k+1}\right)-\frac{E}{k}\right)+(1-x)\cdot\left(v\left(\frac{1}{2}\right)-E\right)$$

$$f(x,x)=x\cdot f(\frac{E}{k},x)+(1-x)\cdot f(E,1-x)$$

According to the replicated dynamic nature of biology evolutionary, a user will replace its strategy with another strategy if its utility is less than the average utility of population. Therefore, the proportion of users that select different strategy will change. The change rate of proportion of user that selects certain strategy is proportional with the range that the user's expectation utility exceeds the average expectation utility. Hence, using replicated dynamic equation, the change rate of proportion of user that selects primary strategy E/k can be expressed as:

$$\frac{dx}{dt} = x[f(\frac{E}{k},x) - f(x,x)]$$
$$= x[f(\frac{E}{k},x) - x\cdot f(\frac{E}{k},x) - (1-x)\cdot f(E,1-x)]$$
$$= x(1-x)[f(\frac{E}{k},x) - f(E,1-x)]$$
$$= x(1-x)\left[x\left(v\left(\frac{1}{2}\right) - v\left(\frac{1}{k+1}\right)\right) + (1-x)\left(v\left(\frac{k}{k+1}\right) - v\left(\frac{1}{2}\right)\right)\right] \quad (7)$$

Replicated dynamic equation (7) can be marked as $dx/dt = F(x)$ for the sake of simplicity. Let $F(x) = 0$, we can obtain the fixed(stable) point of equation (7). There are three stable points at most that can be derived from equation (7).

① $x^* = 0$. Means that all users in population select primary strategy E/k.

② $x^* = 1$. Means that all users in population select mutation strategy E.

③ $x^* = \frac{v(1/2) - v(k/(k+1))}{v(1/2) - v(k/(k+1)) + v(1/2) - v(1/(k+1))}$. Means that the bid game of users will produce an equilibrium of mixed strategy.

The first two stable points mean that users tend to select the same strategy (E/k or E) corresponding to pure strategy equilibrium. The third stable point means that grid users select different strategy according to certain proportion. This point is correspondent to mixed strategy equilibrium. But, the above analysis could not decide which stable point the replicated dynamic process will go. The real stable status must have stability for tiny disturbing. That is to say, if the proportion is away from these stable points, replicated dynamic nature will get the proportion back. This characteristic can be expressed as follows.

$$\begin{cases} F(x) > 0, & 0 < x < x^* \\ F(x) < 0, & x^* < x < 1 \end{cases}$$

Derive the replicated dynamic function with respect to x as:

$$F'(x) = x(2-3x)\left(v\left(\frac{1}{2}\right) - v\left(\frac{1}{k+1}\right)\right) + (1-x)(1-3x)\left(v\left(\frac{k}{k+1}\right) - v\left(\frac{1}{2}\right)\right) \quad (8)$$

Equation (8) can be solved using utility matrix of users. The point satisfied $F'(x^*) < 0$ is the evolutionary stable point of this game.

3.4 Stable Point Discussion

① If $v(1/(k+1)) < v(1/2) < \frac{v(1/(k+1)) + v(k/(k+1))}{2}$, then $x^* = 1$ is evolutionary stable point, and all users select Primary strategy $E\ k$. Replace x in equation (8) with three stable points respectively, only $x^* = 1$ satisfies condition $F'(x^*) < 0$. The replicated dynamic diagram can be shown as figure 2(a).

② If $v(k/(k+1)) < v(1/2) < \frac{v(1/(k+1)) + v(k/(k+1))}{2}$, then $x^* = 0$ is evolutionary stable point. Replace x in equation (8) with three stable points respectively, only $x^* = 0$ satisfies condition $F'(x^*) < 0$. The replicated dynamic diagram can be shown as figure 2(b).

③ If $v(1/2) < v(1/(k+1))$ and $v(1/2) < v(k/(k+1))$, then $x^* = \frac{v(1/2) - v(k/(k+1))}{2v(1/2) - v(k/(k+1)) - v(1/(k+1))}$ is evolutionary stable point. The replicated dynamic diagram can be shown as figure 2(c). After several repeated games, users will select mixed strategy ($x^* E\, k$,$(1-x^*)E$). Intuitively, the proportion of users that select primary strategy E/k is $\frac{v(1/2) - v(k/(k+1))}{2v(1/2) - v(k/(k+1)) - v(1/(k+1))}$, while the proportion of users that select mutation strategy E is $\frac{v(1/2) - v(1/(k+1))}{2v(1/2) - v(k/(k+1)) - v(1/(k+1))}$.

④ If $v(1/2) > v(1/(k+1))$ and $v(1/2) > v(k/(k+1))$, we get $F'(0) < 0$, $F'(1) < 0, F'\left(\frac{v(1/2) - v(k/(k+1))}{2v(1/2) - v(k/(k+1)) - v(1/(k+1))}\right) > 0$. At this time, there are two points satisfy condition $F'(x^*) < 0$. The replicated dynamic diagram can be shown as figure 2(d). After many repeated games, all users will agree to select pure strategy $E\, k$ or E .

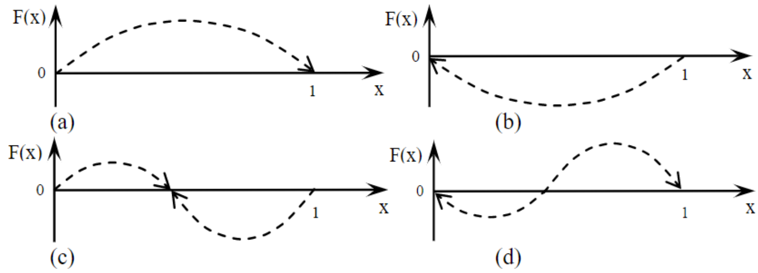


Fig. 1. Replicated dynamic diagram of user population

4 Experiments

4.1 Evolutionary Algorithm

The basic idea of evolutionary game is that a strategy that can obtain preferable utility at this game will be selected by more users at next game. User selects strategy at current game according to the results of previous game. Then, the strategy selection algorithm for user bid in the evolutionary game can be described as:

①At beginning,t =1,initialize utility matrix and related parameters.
②If user selects primary strategy s_0 last time
 if $f(s_0, x_{t-1}) < f(x_{t-1}, x_{t-1})$ user selects mutation strategy s_1;
  $x_t := x_{t-1} - 1/N$;
 else
  user keeps primary strategy s_0 ;
③if user select mutation strategy s_1 last time
 if $f(s_1, x_{t-1}) < f(x_{t-1}, x_{t-1})$ user select primary strategy s_0;
  $x_t := x_{t-1} + 1/N$;
 else user keeps mutation strategy s_1;
④Game plays;
⑤if $(|x_t - x_{t-1}| < 1/N) \&\& (|x_{t-1} - x_{t-2}| < 1/N)$
x_t is evolutionary stable point, $(x_t s_0, (1-x_t) s_1)$ is evolutionary stable strategy, turn to step ⑥;
else
return to step ②;
⑥End.

Intuitively, if the utility of user that select certain pure strategy(primary strategy or mutation strategy) is less than the average utility of population, the proportion of user that select this pure strategy will reduce, and increase otherwise.

4.2 Experiment Setups

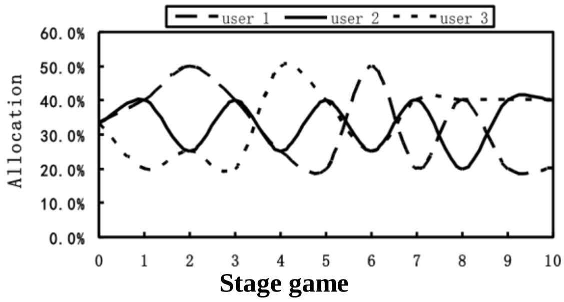
This work simulates a cloud environment based on a Java-based discrete-event simulation toolkit called GridSim [13]. The toolkit provides facilities for modeling and simulating grid resources and grid users with different capabilities and configurations.

The bandwidths of all links are uniformly distributed between 50 and 100 Mbps. Network latency varies from 0.001 to 0.1 s. If resource doesn't receive the user request because of network latency, the user request will be neglected. Resource capacity varies from 2000 to 6000 MIPS. Resource calculates an allocation during an interval of 50 time units to leave enough time for task execution. We model users as many as 25 that are competing for resource. Each user contains all information related to task execution management details such as tasks processing requirements, expressed in MIPS, disk I/O operations, the size of input files, etc. that help in computing execution time of remote resource and the size of output files. Each task length varies from 2000 to 3000 MI. The length of input or output file is set from 100 to 250 MI. User budget varies from 0.5 to 2 $. Task execution cost cannot exceed budget, otherwise user will exit the competition. For the sake of simplicity in analyzing the result, we assume all the users have the same form of valuation function, i.e. $10\ln(1+w_i)$.

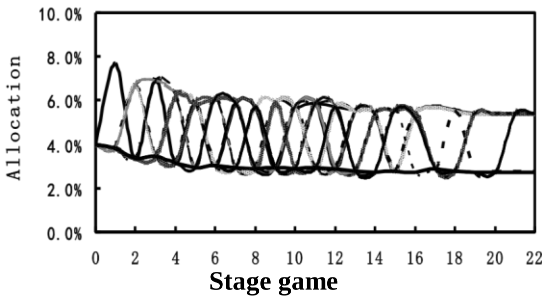
4.3 Experiment Results

First experiment is to study the changing process of resource partition obtained by each user. An evolutionary game is composed of many stage games. The equilibrium portion obtained by each user at certain stage game will change at the next stage game. After a lot of stage games, the equilibrium portion of each user will fix on some value. Hence, the evolutionary game for resource allocation will converge to an equilibrium point.

The X-axis shows the stage games in the evolutionary game. It can be seen from Figure 2 that user changing strategy leads to the change of resource share allocated to the user. Moreover, the allocation portion of a user will increase at the cost of the allocation portion of other user decreases. The reason is that resource is managed by proportional sharing mechanism. In Figure 2(a), the resource allocation portion varies from 20% to 50%, and the game over at the eighth stage game. In Figure 2(b), the resource allocation portion varies from 3% to 8%, and the game over at the twentieth stage game. The effects indicate that more users sharing the resource, less allocation portion obtained by each user. At the same time, converging speed of the evolutionary game will slow down, which matches well with the bounded rationality of users.



(a) 3 users



(b) 25 users

Fig. 2. Changing process of resource share allocated to each user in the evolutionary game

Second experiment is to compare the resource price at stage games under different scale of user population. Resource price is the sum of bids of all users. Users change its bid strategy continuously before reach stable equilibrium of evolutionary game, which leads to the change of resource price. From the result of Figure 3, we can see that resource price increases for certain user population. This is due to the fact that users select primary strategy initially. At this time, resource price is lowest. Resource price will increase because there are users selecting mutation strategy at later stage game. And that resource price will be higher with more users. Resource price is only 2.5$ under 3 users; 10.5$ under 15users; and 18.5$ under 25 users. This effects is consistent with the market mechanism that resource price increases when demand is more than supply.

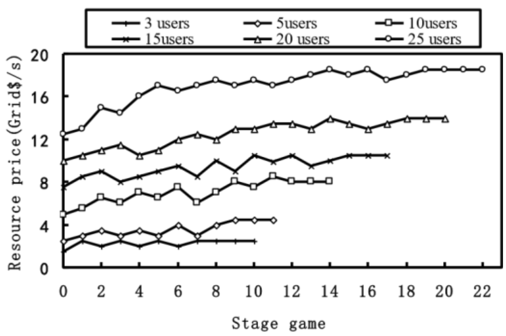


Fig. 3. Replicated dynamic diagram of user population

5 Conclusions

This work addresses resource allocation issues using evolutionary game. Traditionally, the game analysis based on complete rationality of user has been used in resource allocation, but does not take dynamic equilibrium of strategy selection into consideration. In this paper, evolutionary game is presented as a promising method for bounded rational grid users in resource allocation. An evolutionary game model is established to study the behavior of user bid. Using utility matrix and replicated dynamic equation, the evolutionary stable point of proportion in strategy selection can be solved. In particular, the effects of different user evaluation functions on evolutionary stable point are discussed in detail. The evolutionary game method in resource allocation is evaluated. Results show that evolutionary game method could make users adjust their strategies through repeated games to reach stable equilibrium, and thus achieve resource allocation optimization.

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