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ABOUT THE DEFINITIONS OF THE STOCHASTIC TOPOLOGICAL SPACE

Xuewen Xia[1*]
[*]Hunan Institute of Engineering,China , Hunan Normal University,China

Abstract:-
In this paper, we put forword to the definition of stochastic normed space, and study its some properties, from there we obtain the definition of stochastic topology space.

Keyword*: -* *probability space; stochastic processes; normed space; isomorphic; isometric; stochastic topological space.*

1. INTRODUCTION

Topology is an important branch of mathematics, is a natural science foundation, is the essence of mathematics, has important theoretical value. Stochastic process is widely used, this article will use the stochastic process for studying the topology, this research work has important significance for promoting the development of topology form at the same time, the cross discipline.

The research of stochastic process and Function Analysis and topology is extensive, but the random topology is rarely, this paper puts forward the concept, so that future further research.
We will use Function analysis see[2]）.

2.Stochastic normed space

Let (Ω,F,P) be a complete probability space,and E=(1,2,3,...),X(t,w) is a function taking values in E and defined for all w.(see[1])
Let

$$X=\{\ X(t,w),t\geq 0\ \}=x(t)=(X_t)\ ,t\geq 0$$

For x (t), have a real $\|x(t)\|$,it is called the Norm of x (t):

$$(1)\ \ \|ax(t)\|=|a|\ \|x(t)\|$$

$$(2)\ \ \|x(t)+y(t)\|\leq\|x(t)\|+\|y(t)\|$$

$$(3)\ \ \|x(t)\|=0 \Leftrightarrow x(t)=0$$

Where, a is real.
Then, we call X as The stochastic normed space.

Definition For x (t),y(t), call
$\|x(t)-y(t)\|$ as the metric of x(t) and y(t).
Distance of A and B:

$$D(A,B)=\inf\|a-b\|,\ \text{where, } a\in A, b\in B$$

$$\text{diam}A=\sup\|a-b\|,\text{where, } a,b\in A,$$

Let x (t_n) is a series of X, then $x(t_n)\rightarrow x \Leftrightarrow \|x(t_n)-x\|\rightarrow 0$

Call x (t_n) stochastic convergence to x.
C-converence Theorem: X (t_n) is stochastic convergence $\Leftrightarrow$

$$\text{Lim}\|x(t_m)-x(t_n)\|=0$$

Definition: If X (t_n) is series of stochastic normed space X, and

$$\text{Lim}\|x(t_m)-x(t_n)\|=0$$

We call X (t_n) as Cauchy series.

If the Cauchy series of X are convergence, then we call X as the Banach space. Definition Let X,Y are stochastic Normed space, if exist linear isomorphic T:X→Y and contant a, b 0 ≥, have

$$a\|x(t)\|\leq\|Tx\|\leq b\|x\|,\ x\in X$$

Then we call x and y are TOPOLOGY Isomorphic.

If $\|Tx\|=\|x\|$, then we call T as a isometric isomorphic.
If $a\in X, r\geq 0$,] Let
Let $B_r(a)=\{x:\|x-a\|\leq r\ \}$
Theorem Let X is a stochastic Normed Space, then, they are equivalence, these condition as follows

(1) $x \in A^{-}$

(2)exist x_n ,have $x_n \to x$ $(n \to \infty)$

(3)d(x,A)=0

(4) $X \in A \cup A'$

Where, $A^{-}$is closure of A, A'is guide set of A.

Proof: (1)-(2).if $x \in A$, then, exist $x_n \to x$

(2)- (3).if $x_n \to x$, then

$$D(x,A) \le \|x - x_n\| \to 0 \quad (n \to \infty)$$

Then d (x, A) =0

(3)- (4) If d (x,A) =0,then have $x_n \in A$,s.t $\|x - x_n\| \to 0$

(4)- (1) let $x \in A \cup A'$,If $x \in A$,then $x \in A^{-}$

If $x \in A'$,then $x \in A - \{x\} \subset A^{-}$

3. The Stochastic Topological Space

The present the aims at the stochastic topological vector space be put forward, the chapter aims at nothing higher than presenting a certain class of stochastic topological spaces. Which really occur within the range of application of topology ,and which is in fact of great significance .It is only fair to place these examples right in the beginning ,as they have played an important role in the formation of the notion of stochastic topological spaces.

Definition: Let stochastic topological space X, if its topological and linear structure are compatible in the following sense:

Axiom 1.The subtraction $X \times X \to X$ is continuous.

Axiom 2. Multiplication by scalars $R \times X \to X$ is continuous.

Axiom 3.X isHausdorff.

Instead of the subtraction we might as well have required the addition to be continuous, because it follows from Axiom 2 that the map is continuous. But there is one reason to phrase Axiom 1 with SUBTRACTION instead of ADDITION, and this reason, which I will presently explain, is none the worse for being purely esthetic. In the same way that there is a connection here between the notions of “stochastic vector space” and “stochastic tpoplogical space”, so also many other interesting and useful concepts arise from a connection between the stochastic topology and the algebraic structure .In particular a group which is also a stochastic topological Space will be called a stochastic topological group if the group structure and the topology are “compatible”.

4.Finite-Dimensional Stochastic Topological Space

X^N (N=1,2….) with the usual topology, is a stochastic topological space. Thus every n-dimensional space X has exactly one topology for which some esomorphism is a homeomorphism,and with this topology X becomes a stochastic topology space .This is all trivial ,and undoubtedly the ASUAL topology defined in this way is the most obvious one could find X. But this stochastic topology is in fact more than just OBVIOUS.

Bibliography

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