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GLOBAL METHOD COMBINED WITH FLOQUET MODAL ANALYSIS TO MODEL AN ANTENNA BASED ON RECTANGULAR WAVEGUIDE ARRAY

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Abstract:-
In this study, we propose a new formulation based on a global method to model a rectangular waveguide array as antenna. Our method consists to combine the method of moment (MoM), the generalized equivalent circuit method (GECM) and Floquet modal analysis. Looking for rigor, simplicity and rapidity, we repose always on the simple discontinuity in an open rectangular waveguide. So, we extended our pervious works to study in first step an infinite rectangular waveguides array and we achieve with a finite one. For validation purpose, we consider two rectangular waveguides and we compered it whit the same structure studied by direct method in our previous study. Obtained numerical results are presented and discussed. A good agreement is shown and the boundary conditions are verified.

Keywords: - *Modeling, Antenna, Array, waveguide, Global Method, Floquet, GECM*

1. INTRODUCTION

The radiating characteristics of a single antenna are insufficient for wireless telecommunications and radar applications. For this purpose, antenna arrays are generally used to customize the radiation patterns by acting on the phases or modules relating to the elementary antennas by using beam-forming networks [1,2].

In satellite and radar applications, waveguide array is usually used as antennas [3,4]. Rectangular waveguides are widely used than circular waveguides because they are easier to manipulate and they offer a lower cross polarization component [5]. In our previous works, we have proposed a new rigorous approach to model a finite open ended waveguide array radiating in free space [6,7]. This formulation allows to use the MoM-GEC method to model a single waveguide and we extend this formulation to study a twice waveguides radiating in free space.

The study of finite waveguides array is useful both for studying its characteristics and for determining the edge effects for nearest elements or at the edges of very wide array. In [8], the mutual coupling problem in a waveguide array radiating in the free space is expressed in terms of an aperture half-space admittance matrix and scattering parameters between the array elements. An important characteristic of antenna array is to manipulate the antenna radiation pattern by acting on the array element excitation's [9]. In many previous works, rectangular waveguide arrays are used to enhance radiation properties. For this, some amplitude distribution techniques are used, such as uniform amplitude distribution [10], parabolic amplitude distribution, and circular amplitude distribution [11]. These three distributions are used to generate the array pattern needed for wireless and radar communications.

The radiated far field at a point in the space is the sum of the contribution of each array element's. In the case of identical array elements, the antenna array radiation pattern is calculated by multiplying the radiation pattern of single antenna by the array factor. This concept is called pattern multiplication [12,13].

The main objective is to find the appropriate antennas configuration, the antenna dimensions and the excitation distribution to produce a radiation pattern with low side lobes and narrow beam width. [14]

However, the main problem of the direct method is the complexity of the computation which increases with the array elements number. In fact, the needed test function number increase quickly with the array elements number.

In this paper, we propose a rigorous formulation based on MoM-GEC method combined to Floquet's modal analysis to model a rectangular open ended waveguide array radiating in free space. For a validation purpose, the proposed formulation is applied to a simple waveguide array formed by two rectangular waveguides. Obtained results show a good agreement with the direct method.

2. An infinite rectangular waveguides array radiating in free space

As shown in figure 1(a), we consider the same structure studied in our previous work [7]. A real waveguide (RG) with electric wall (EW) opened in another waveguide (SG) aims to simulate the free space. But in this study, the SG is composed by periodic wall (PW). The two waveguides are tied by a transversal metallic wall. In figure 1(b), we present the relative equivalent circuit model.

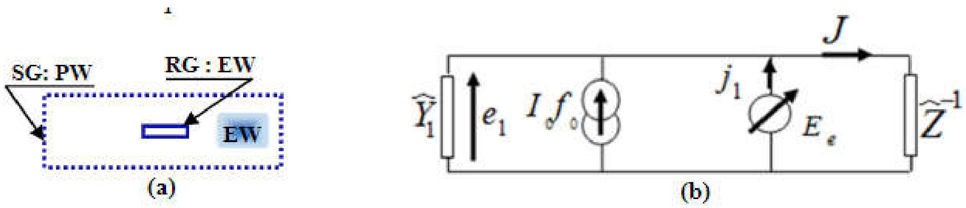


Figure 1. (a) An infinite rectangular waveguides array; (b) the relative equivalent circuit.

The excitation is brought back to discontinuity plan as a modal source of current. Its value is the current density of the fundamental mode I f , and its internal admittance is $\hat{Y}_1$' which represents the evanescent modes' contribution of RG. $\hat{Y}_1$' is given by the formula:

$$\hat{Y}_1 = \sum \left| f_{pq} \right\rangle y_{pq} \left\langle f_{pq} \right| \quad (1)$$

where f_{pq} is the RG modal basis without the fundamental f_0 and y_{pq} is the mode admittance of each f_{pq}. The voltage at terminals of this source is e_1, which is its dual greatness.

E_e is the virtual voltage source defined in discontinuity plan, and 1j is the current flowing it. E_e is the unknown problem, and it is expressed as a serial of test functions g_p weighted by unknown modal amplitudes x_p :

$$E_e = \sum_p x_p g_p \tag{2}$$

Next, we apply the Galerkin method wherein the test functions are same as modal basis functions. The modes contribution of SG is expressed in the discontinuity plane by the admittance operator $\hat{Z}^{-1}$ which is given by the following formula.

$$\hat{Z}^{-1} = \sum \left| F_{mn} \right\rangle y_{mn} \left\langle F_{mn} \right| \tag{3}$$

Where F_{mn} the SG is modal basis and y_{mn} is the relative mode admittance.

Based on the equivalent circuit shown in Figure (1), the integral equations associated with the problem can be easily derived by applying generalized Kirchhoff laws:

$$\begin{cases} J = \hat{Z}^{-1} E_e \\ J = -\hat{Y}_1 E_e + I_0 f_0 + j_1 \end{cases} \tag{4}$$

Using the equations system (4), we can write:

$$\hat{Z}^{-1} E_e = -\hat{Y}_1 E_e + I_0 f_0 + j_1 \tag{5}$$

Then, we can deduce the current j_1:

$$j_1 = (\hat{Y}_1 + \hat{Z}^{-1}) E_e - I_0 f_0 \tag{6}$$

$$\begin{pmatrix} e_1 \\ j_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \hat{Z}^{-1} + \hat{Y}_1 & -1 \end{pmatrix} \begin{pmatrix} E_e \\ I_0 f_0 \end{pmatrix} \tag{7}$$

Applying the Galerkin's method leads to the following system of equations:

$$\begin{cases} V_0 = \left\langle f_0 \middle| E_e \right\rangle = \sum_q x_q \left\langle f_0 \middle| g_q \right\rangle = A^t X \\ -I_0 \left\langle g_p \middle| f_0 \right\rangle + \sum_q \left\langle g_p \middle| (\hat{Y}_1 + \hat{Z}^{-1}) g_q \right\rangle x_q = 0 \end{cases} \tag{8}$$

This system can be rewritten as:

$$\begin{cases} A^t X = V_0 \\ -A I_0 + [y] X = 0 \end{cases} \tag{9}$$

Where A is the excitation vector, and $[y]$ is the admittance matrix:

$$A = \left\langle f_0 \middle| g_p \right\rangle \tag{10}$$

$$[y] = \begin{bmatrix} \left\langle g_{10} \middle| (\hat{Y}_1 + \hat{Z}^{-1}) g_{10} \right\rangle & \cdots & \left\langle g_{10} \middle| (\hat{Y}_1 + \hat{Z}^{-1}) g_{pq} \right\rangle \\ \vdots & \ddots & \vdots \\ \left\langle g_{pq} \middle| (\hat{Y}_1 + \hat{Z}^{-1}) g_{10} \right\rangle & \cdots & \left\langle g_{pq} \middle| (\hat{Y}_1 + \hat{Z}^{-1}) g_{pq} \right\rangle \end{bmatrix} \tag{11}$$

Solving the above system (9) we get:

$$X = [y]^{-1} A I_0 \tag{12}$$

Taking into account relations (8) and (12), we deduce the impedance matrix Z:

$$Z = A^t [y]^{-1} A \tag{13}$$

The scattering matrix $[S]$ can therefore be expressed using the reduced impedance *z* and the identity matrix:

$$[S] = (z - I)(z + I)^{-1} \tag{14}$$

3. A finite rectangular waveguides array radiating in free space

Our study structure is a periodic array composed by nine rectangular waveguides, called NG (Network-Guide), adiating in free space. This latter is simulated by another rectangular waveguide, called SG (Space-Guide), with large imensions. All structure is linked by a transversal electrical wall and (Figure2).

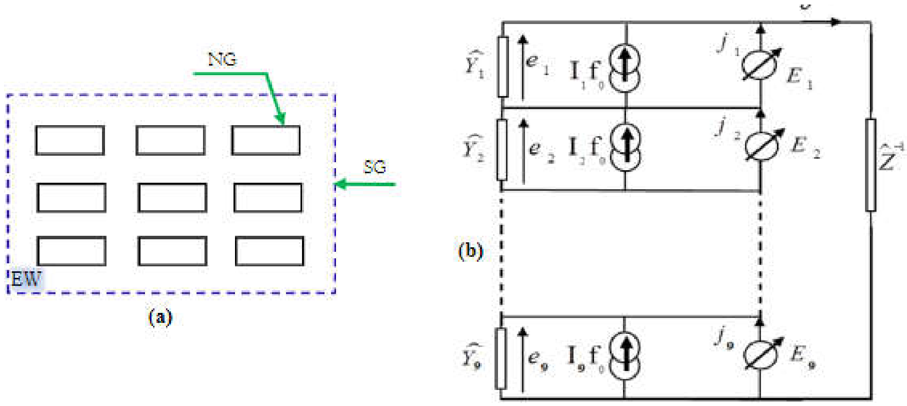


Figure 2. (a) A finite rectangular waveguides array (b) the relative equivalent circuit.

This formulation is very different of a unit cell in terms of complexity. Indeed, increasing the number of array antennas will made the direct approach is difficult to contemplate.

For this, we propose to consider a structure studied in Figure (1) as a unit cell and we introduce the Floquet modal analysis to apply it on this cell. The Floquet method permit to duplicate several times a unit cell on the x and y axes to form an array as desired. Then, the cell (i,s), in Figure (3), is considered as a translation of the cell (0,0) in space domain followed by another in the modal domain and expressed in dependence of the Floquet modes phases (α, β). So, if we assume that:

$(i,p)\in\mathbb{Z}^2$ And comprised within the range $\left[-\frac{N_x}{2},\frac{N_x}{2}-1\right]$

$(s,q)\in\mathbb{Z}^2$ And comprised within the range $\left[-\frac{N_y}{2},\frac{N_y}{2}-1\right]$

N_x and N_y are the numbers of array elements in the x and y directions respectively
L_x and L_y are the lengths of the array respectively according to the axes x and y.

α_p, b_a will have the following forms:

$$\alpha_p=\frac{2\pi p}{L_x} \tag{15}$$

$$\beta_q=\frac{2\pi q}{L_y} \tag{16}$$

Where L_x and L_y are given by:

$$L_x=N_x d_x \tag{17}$$

$$L_y=N_y d_y \tag{18}$$

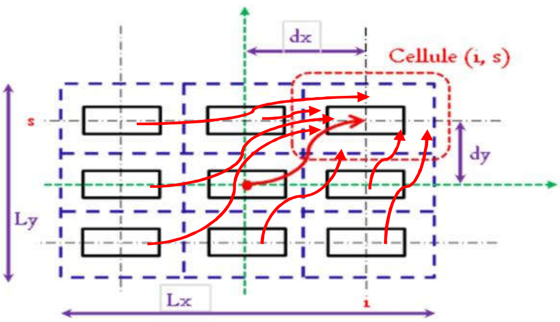


Figure 3. A finite rectangular waveguide array.

So, the electric field at the position of cell (i, s) can be written as the superposition of the fields associated with the Floquet modes (spectral domain). And we can consider this decomposition as the transformation of the electric field of the space domain to modal domain (Figure 3). Mathematically this transformation is the inverse Fourier transform (19).

Similarly, the radiating field of phase shift (α,β) can be written as a sum of the radiated energy of this field on all cells. This summation can be seen as the Fourier transform of the electric field of the modal domain in space domain (20).

$$E(i,s)=\frac{1}{\sqrt{N_x N_y}}\sum_{p,q}\tilde{E}_{\alpha_p,\beta_q}e^{j\alpha_p(idx)}\cdot e^{j\beta_q(sdy)} \tag{19}$$

$$\tilde{E}_{\alpha_p,\beta_q}=\frac{1}{\sqrt{N_x N_y}}\sum_{i,s}E(id_x,sd_y)\cdot e^{-j\alpha_p(id_x)}\cdot e^{-j\beta_q(sd_y)} \tag{20}$$

It has been demonstrated [15,16] that these transformations lead to other expressions emphasizing the coupling terms established between array elements such as the Z, Y and S parameters.

$$\left[Z_{i,s}\right]=TF^{-1}\left[\tilde{Z}_{\alpha_p,\beta_q}\right]TF \tag{21}$$

$$\left[Y_{i,s}\right]=TF^{-1}\left[\tilde{Y}_{\alpha_p,\beta_q}\right]TF \tag{22}$$

$$\left[S_{i,s}\right]=TF^{-1}\left[\tilde{S}_{\alpha_p,\beta_q}\right]TF \tag{23}$$

Here, $\left[\tilde{Z}_{\alpha_p,\beta_q}\right]$ is a spectral representation of the diagonal matrix that concatenates the intrinsic input impedance associated with a cell Floquet modes of the array. In our case, it is written as follows:

$$\left[\tilde{Z}_{\alpha_p,\beta_q}\right]=\begin{bmatrix} \tilde{Z}_{\alpha_{-1},\beta_{-1}} & 0 & . & . & . & . & . & . & 0 \\ 0 & \tilde{Z}_{\alpha_{-1},\beta_0} & . & & & & & & . \\ . & . & \tilde{Z}_{\alpha_{-1},\beta_1} & . & & & & & . \\ . & & . & \tilde{Z}_{\alpha_0,\beta_{-1}} & . & & & & . \\ . & & & . & \tilde{Z}_{\alpha_0,\beta_0} & . & & & . \\ . & & & & . & \tilde{Z}_{\alpha_0,\beta_1} & . & & . \\ . & & & & & . & \tilde{Z}_{\alpha_1,\beta_{-1}} & . & . \\ . & & & & & & . & \tilde{Z}_{\alpha_1,\beta_0} & 0 \\ 0 & . & . & . & . & . & . & 0 & \tilde{Z}_{\alpha_1,\beta_1} \end{bmatrix} \tag{24}$$

4. Numerical Results

4.1. An infinite rectangular waveguides array

We determine in this case the reflexing parameter and the normalized electric field for $d=2\lambda$.

$$\left|S_{11}\right|=0.3147 \tag{27}$$

In Figure 4, we present the normalized electric field distribution on discontinuity plan, and we can verify that the coupling is very low especially on the x axis. This can be explained by the fact that the fundamental mode TE10 which represents the source of the electric field discontinuity has no component along the x axis. The allure of the electric field was not affected by the periodic walls, and the boundary conditions are verified; the field is maximum at the center of the opening and it vanishes on the metal part.

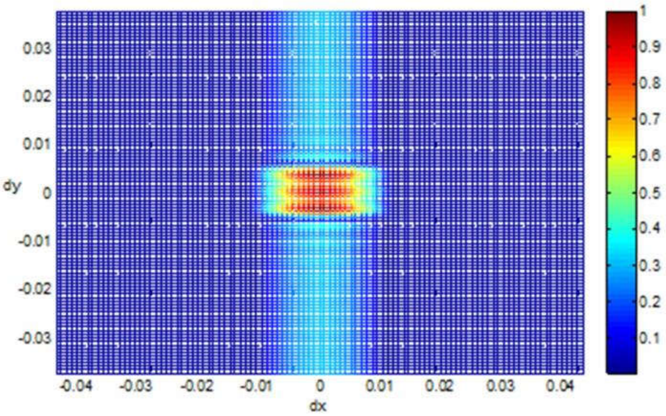


Figure 4. Normalized electric Field distribution on discontinuity, d = 2λ, f = 9.33GHz

4.2. A finite rectangular waveguides array

For the finite network, we propose to make a parametric study based on coupling distances. So, we choose three distances: lower than wavelength λ, in the order of λ and upper than λ.

4.2.1. Scattering parameters

When we determine the S parameters matrix for the different coupling distances, we remark that there are values that are repeated, and their positions in matrix do not change regardless of the coupling distance. Then, to better understand the meaning of this, we stained in this usual matrix (Figure 5. (a)) each set of these values in the same color. So, if we consider the initial cell, we can represent the diffraction parameters as follows (Figure 5. (b)), all the others are similar.

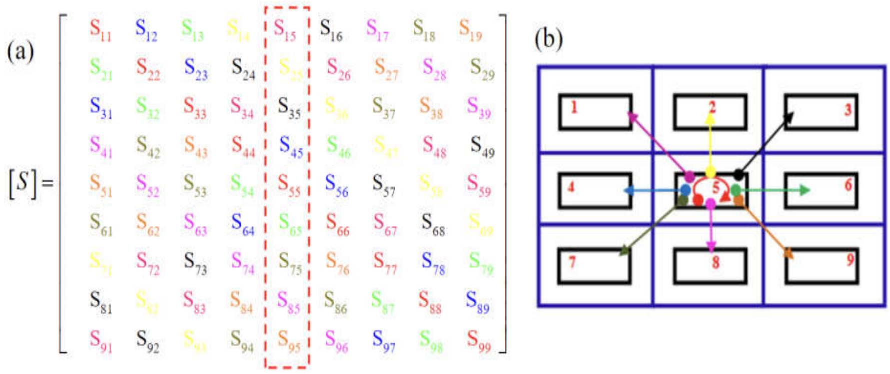


Figure 5. (a) Usual Scattering matrix; (b) Schematic representation of S parameters.

In the Table 1, we represent the squares of values of nine sets of diffraction parameter as a function of the coupling distance d. These physical magnitudes present the percentages of powers relating to these parameters with respect to the power initially issued.

Table 1. Scattering parameters as a function of coupling distance d.

d (λ)	$\lvert S_{55}\rvert^2$	$\lvert S_{45}\rvert^2$	$\lvert S_{65}\rvert^2$	$\lvert S_{25}\rvert^2$	$\lvert S_{85}\rvert^2$	$\lvert S_{75}\rvert^2$	$\lvert S_{35}\rvert^2$	$\lvert S_{95}\rvert^2$	$\lvert S_{15}\rvert^2$
2	0,0661	0,0801	0,0546	0,1285	0,0306	0,0058	0,0135	0,0523	0,0115
1	0,0780	0,1000	0,0969	0,1526	0,0938	0,0101	0,0260	0,0541	0,0293
0.1	0,1418	0,1054	0,1000	0,1643	0,1095	0,0181	0,0370	0,0587	0,0324

To understand the physical meaning of these parameters we define three follow types of power:

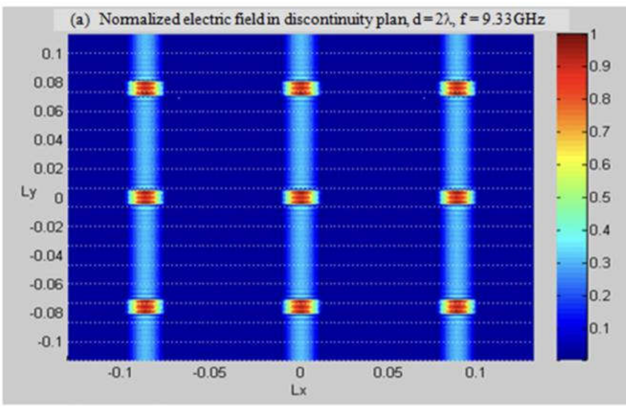
Table 2. Comparison of self-reflected power, radiated power and coupled power, according to the coupling distance d.

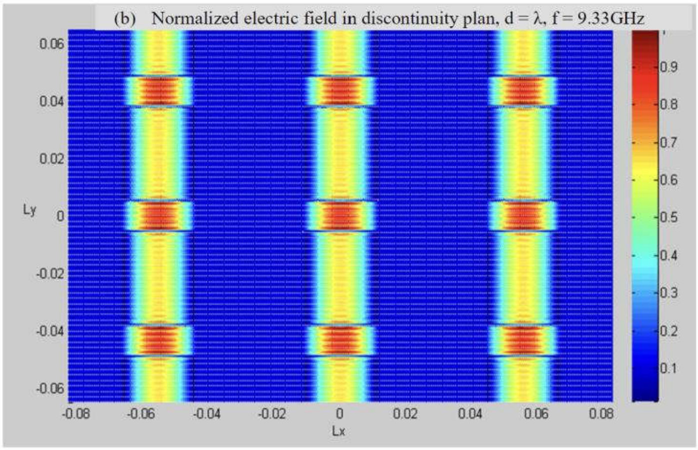
d(λ)	SP	CP	RP
2	0.0661	0.3769	0.5570
1	0.0780	0.4628	0.4592

4.2.2. Electric field distribution

We represent the electric field distribution on the discontinuity surface for the three cases respectively.

And we can notice that the conditions to limit its verified except for low coupling distance, where the coupling is so intense that the fields overflow the openings.





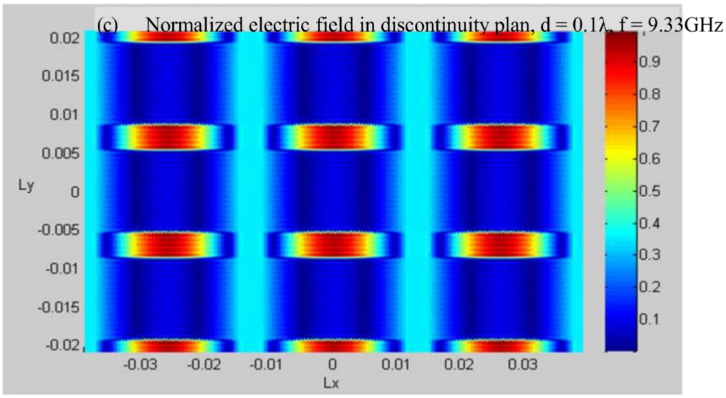
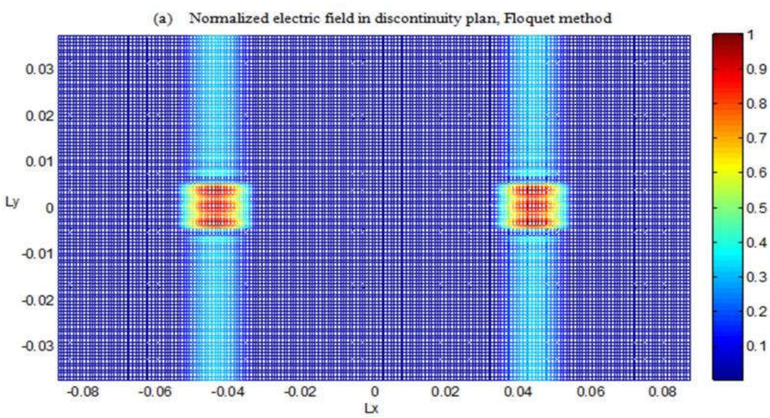


Figure 6. Normalized electric Field distribution on discontinuity plan for f=9.33GHz
(a) d=2λ; (b) d=λ; (c) d=0.1λ

4.3. Validation

To validate this new formulation based on Floquet modal analysis, it's proposed to apply it into a a dual waveguide network in order to compare it with the direct method used in previous work [6].

As shown in Figure 8, first, we can confirm that there is a good agreement in term of the boundary conditions. Second, we can verify the presence of the same behaviours of electric field in waveguides modeling the free space for the two methods. Third, we can validate that the maximum of electric field distribution is concentrated in the network waveguides as it should be. However, we can remark that the distributions of electric fields in the network waveguides have not a same appearance. Indeed, it's due to the use of different methods. Other works should be held in order to remedy to this result.



(b) Normalized electric field in discontinuity plan, Direct method

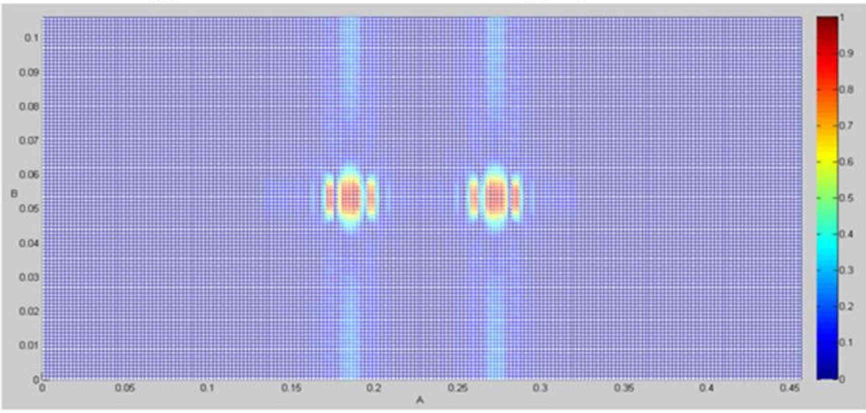


Figure 7. Normalized electric Field distribution d=2λ and on discontinuity plan for a network of two waveguides for a coupling distance operational frequency *f* = 9.33GHz (a) Method based on Floquet modal analysis; (b) Direct Method.

5. Conclusion

In this work, we proposed a new formulation based on the combination of MoM-GEC method and Floquet modal analysis to model a finite network of rectangular waveguide radiating in free space.

We conceive an infinite array by using the property of periodic walls with a single waveguide. Then, we consider this structure as a unit cell, and we applying the Floquet modal analysis in order to model the finite array. Later, we apply a parametric study to understand how the electric field and coupling behaves based on coupling distance.

For validation purpose, first, we compared the scattering parameters with the results published in previous work, and verified the boundary conditions and good agreement is shown. Secondly, we compared the distribution of electric field in the discontinuity plane for a two waveguides array by two formulations, a direct one and Floquet modal analysis one, and good agreement is shown.

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