

MODIFIED RESPONSE SPECTRUM ANALYSIS FOR CONTROLLED STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION.

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Abstract:-

One case of the earthquake analysis is the response spectrum analysis. Where the structure is designed to be protected by earthquakes which are represented from design spectrum. However, very often the real earthquake that applied to structure exceeds the design spectrum. In that case the ductility of structure and the demand capacity are the line of defence to face that situation. As a result damage will occur in structure and the cost of rehabilitation is unavoidable. An alternative direction which is proposed in this paper is to design structure, equipped by control devices, capable to resist the incoming earthquake and remaining in elastic range and thus without damage. The idea is that once the response spectrum of the incoming earthquake is higher than the design spectrum at the eigenperiods of the structure the control devices will be activated, in order of milliseconds, and will change the period of structure. In that case the structure will avoid the "resonance" with the incoming earthquake and its response spectrum will lie lower than the design spectrum at the new eigenperiods of the structure. From the numerical results it is shown that the above control strategy is efficient in reducing the response of building structures, with small amount of required control power.

Keywords: - Response spectrum, design spectrum, structural control, earthquake engineering.

1 INTRODUCTION

Structural control applications of seismically excited buildings have gain considerable attention in last decays. The research and application of active control to civil engineering structures include analytical studies and experimental verifications. The works of Housner et al, [1], Kobori et al, [2], and Spenser et al, [3], are the most representative. Several well-established algorithms in control engineering have been introduced to control structures, such as optimal control LQR or LQG, Yang, [4], Abdel-Rohman et al, [5], sliding mode control, Yang et al, [6], H2 and H ∞ , Kose et al, [7], Zacharenakis et al, [8]. The most suitable algorithms for structural applications and the practical considerations that should be taken into account were described by Soong, [9]. Pole allocation control algorithms have been studied extensively in the general control literature, Sage et al, [10], Brogan, [11], Ogata, [12], Kautsky and Nichols, [13]. Applications of the algorithm in structural control can be found in the work of Meirovitch, [14], Soong, [9], Martin et al, [15], Utku, [16], Wang et al, [17]. The pole placement is a well-known, classical control algorithm that estimates the feedback matrix so that the system will have poles (eigenvalues) that are pre-decided by the designer. Successful application of the algorithm requires judicious placement of the closed-loop eigenvalues on the part of the designer, as well as a good understanding of the uncontrolled modal structural behaviour.

In this paper the authors suggest a control algorithm taking into account the earthquake signal and its frequency components compared to the design spectrum of structure. The algorithm compares the two spectrums, the earthquake and the design spectrum and identify the region where the acceleration of the earthquake spectra is higher to the design spectra. If the eigen-periods of the structure are lying at this region then a new position of eigen-periods are chosen and the suitable control force is calculated based on pole allocation algorithm. Otherwise no control action is required.

2 Structural CONTROL BASED ON NON-RESONANCE THEORY

Structural control based on non-resonance strategy consists of two phases: First, the frequency characteristics of the excitation signal should be recognised on line. Especially in the case of earthquake signal where is non resonance signal and its characteristics are chance during the time. Such a case is shown in Fig. 1.

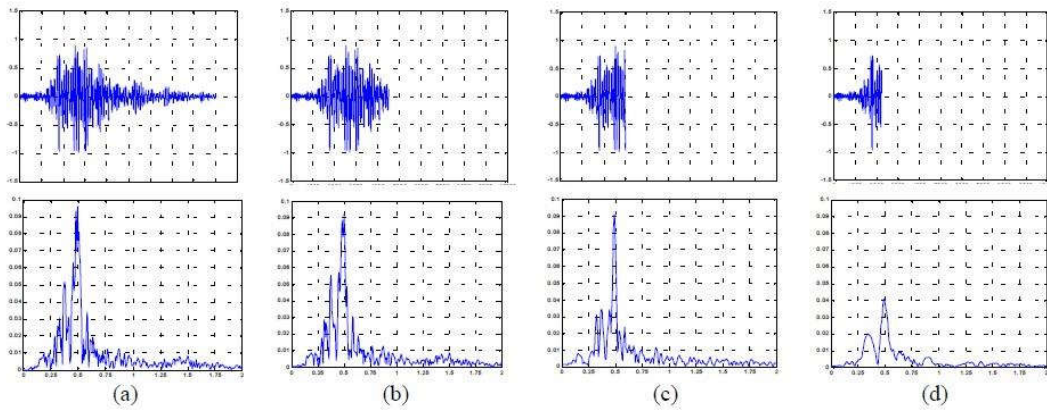


Figure 1 - The acceleration records and corresponding spectra of (a) the whole signal of the Mexico earthquake, (b) 1/2 of the signal, (c) 1/3 of the signal and (d) 1/4 of the signal.

Secondly the elastic and the design spectrum which was used to design the structure should be compared to the earthquake response spectrum. From this comparison may the earthquake spectrum exceed the design spectrum or not. When the design spectrum is above of earthquake spectrum then no control action is required and the structures will response according to its design and no damages are expected. However, it is frequently observed that there are a regions, $a_{e,i}$, of periods, where the response spectrum of an earthquake may exceed the design spectrum, as shown in Fig. 2. In the case where the response spectrum of earthquake is higher than the design spectrum and the structure has its frequencies at this region then from the conventional design the structure due to its ductility will resist and absolved the high earthquake energy but some damage according to the capacity design are expected. In this case, the control action is to force the structure to have an “equivalent” dynamic characteristic (eigen-periods) out of the above regions. The schematic representation of this action is shown in Fig. 3.

The equation of motion of a controlled structural system with n degrees of freedom subjected to an earthquake excitation a_g the state space approach is:

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}_g a_g + \mathbf{B}_f \mathbf{F} \quad (1)$$

The matrixes \mathbf{X} , \mathbf{A} , \mathbf{B}_g , \mathbf{B}_f are given by

$$\mathbf{X} = \begin{bmatrix} \mathbf{U} \\ \dot{\mathbf{U}} \end{bmatrix}_{2n \times 1}, \quad \mathbf{A} = \begin{bmatrix} \mathbf{O} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}_{2n \times 2n}, \quad \mathbf{B}_g = \begin{bmatrix} \mathbf{O} \\ -\mathbf{E} \end{bmatrix}_{2n \times 1}, \quad \mathbf{B}_f = \begin{bmatrix} \mathbf{O} \\ \mathbf{M}^{-1}\mathbf{E}_f \end{bmatrix}_{2n \times 1} \quad (2)$$

where \mathbf{M} , \mathbf{C} , \mathbf{K} denote the mass, damping and stiffness matrices of the structure, respectively, \mathbf{F} is the control force matrix and \mathbf{E} , \mathbf{E}_f are the location matrix for the earthquake and the control forces on the structure.

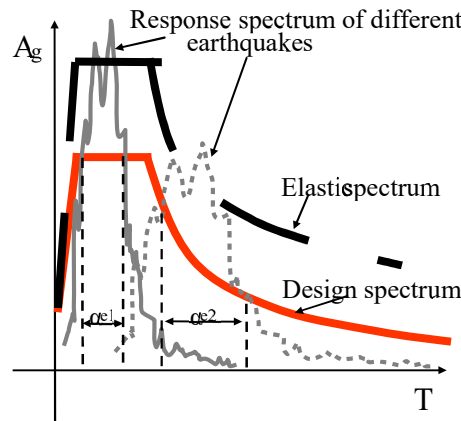


Figure 2 - Elastic, design and response spectrum of earthquake, the width of frequencies α_{ei} where the response spectrum of earthquake exceeds the design spectrum.

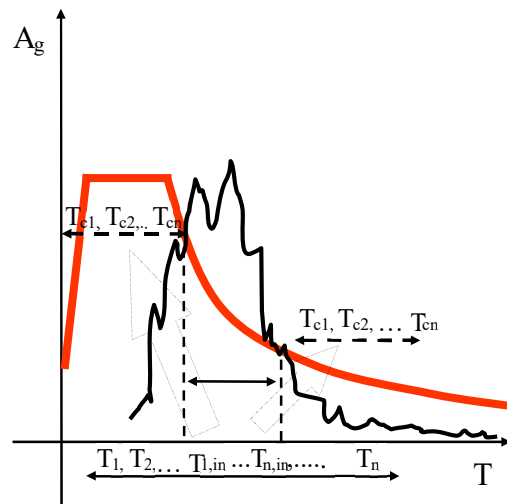


Figure 3 - The region of periods $(T_{1,in}, \dots, T_{n,in})$, where the earthquake response spectrum exceeds the design spectrum, the eigen-periods of uncontrolled structure, $T_1, T_2, T_{1,in}, \dots, T_{n,in}, \dots, T_n$ and the new location of the eigen-periods of the controlled system $T_{c1}, T_{c2}, \dots, T_{cn}$.

The eigenmodes, the eigenperiods T_i or the corresponding eigenfrequencies f_i and the damping ratio ξ_i of the uncontrolled system are obtained from the solution of the following eigenvalue problem:

$$\begin{aligned} \left[\mathbf{K} - \omega^2 \mathbf{M} \right]_{n \times n} \Phi = 0 \quad T_i = \frac{2\pi}{\omega_i}, \quad f_i = \frac{\omega_i}{2\pi}, \quad i = 0, \dots, n-1 \quad \begin{aligned} \mathbf{C}_n &= \Phi_n^T \mathbf{C} \Phi_n \\ \mathbf{M}_n &= \Phi_n^T \mathbf{M} \Phi_n \\ \xi_i &= 2\mathbf{C}_n \mathbf{M}_n \omega_n \end{aligned} \end{aligned} \quad (3)$$

The eigenvalues of the system are obtained directly from the eigenvalues of matrix \mathbf{A} , as indicated by the equation:

$$\det[\lambda \mathbf{I} - \mathbf{A}] = 0 \rightarrow \lambda_i \quad \lambda_i = -2\pi f_i \xi_i \pm j 2\pi f_i \sqrt{1 - \xi_i^2} \quad (4)$$

It is assumed that the control force \mathbf{F} is determined by linear state feedback:

$$\mathbf{F} = -\mathbf{G}_1 \mathbf{U} - \mathbf{G}_2 \dot{\mathbf{U}} = -\mathbf{G} \mathbf{X} \quad (5)$$

\mathbf{G} is the gain matrix which is calculated according to the desired eigenvalues (poles) of the controlled system. Replacing the force \mathbf{F} into equation (1), the closed loop (controlled) system becomes:

$$\dot{\mathbf{X}} = (\mathbf{A} - \mathbf{B}_r \mathbf{G})\mathbf{X} + \mathbf{B}_r \mathbf{a}_g \quad (6)$$

The new eigenvalues λ_{ic} of the controlled system satisfy the following equation:

$$\det[\lambda \mathbf{I} + \mathbf{B}_r \mathbf{G} - \mathbf{A}] = 0 \quad (7)$$

The state feedback design consists of selecting the gain matrix \mathbf{G} so that the roots (eigenvalues) of equation (7) are at the desired locations. The desired eigenvalues λ_{ic} of the controlled system also satisfy the equation:

$$(\lambda - \lambda_{ic})(\lambda - \lambda_{ic}) \dots (\lambda - \lambda_{ic}) = 0 \quad (8)$$

The controllable structural system can always be forced to have the desired eigenvalues by choosing the gain matrix \mathbf{G} in such a way that equation (7) is identical to equation (8). The above simple and well-known procedure is not applicable for higher degree of freedom systems, where some other techniques are used that are implemented in software engineering [Kautsky, et al, 1985, Laub et al, [18]. Even though a controllable system can always be forced to have the desired eigenvalues, however, this may not be practical, because the control gain \mathbf{G} would be too large and a lot of energy would be required, or because the range of values of the feedback gain would render the system oversensitive to noise or to modeling errors when the control law is implemented on the reconstructed state from an observer. These robustness aspects are extremely important and dominate the controller design, so the successful application of the eigenvalue assignment (pole placement) algorithm requires correct selection of the controlled system eigenvalues from the part of the designer. Also, the existence of time delay of the system, which introduces infinite number of poles in the controlled system, requires a good judgment of location of poles of the controlled system.

In this paper the selection of the controlled system eigenvalues is based on the comparison of the response spectrum of the incoming earthquake signal and to the design spectrum which was used for designing of structure. The incoming earthquake signal is analyzed and its response spectrum is recognized. As response spectrum of an initial part of the incoming signal is recovered is compared to the design spectrum and the regions of periods ($T_{1,in} \dots T_{n,in}$), where the response spectrum is higher than the design spectrum are recognised. If the structure has its eigen-periods, T_i , out of this region then no control actions is required. If the important eigen-periods of structure are inside of these regions then a decision of place those eigen-periods out of these regions is taken. Based to the new desired locations of eigen-periods, T_{ic} , the new eigen values of the controlled structure λ_{ic} , are calculated based on equation (9). The eigen-periods of the uncontrolled structure, ($T_{1,in} \dots T_{n,in}$), should be moved left or right and placed out of the regions where the response spectrum of the earthquake is higher than the design spectrum, as shown in Fig. 3.

$$\lambda_{ic} = -\frac{2\pi\xi}{T_c} \pm j \frac{2\pi}{T_c} \sqrt{1-\xi^2} \quad (9)$$

From Fig. 3 it is seen that it is not necessary to move all the eigen-periods of the uncontrolled structure, just only those, which are important (its modal mass contribute up to 90% of the mass of structure) and located inside the critical region, a_e , where the response spectrum of earthquake exceeds the design spectrum. the limits of critical region, a_e , are the lower period T_l and the higher period T_h . How far from the critical region, a_e , of the moved eigen-periods would be located, depends on previously selected design parameters λ_{ic} , as indicated by equation (10). As shown in Fig. 3, the moved eigen-periods, ($T_{1,in} \dots T_{n,in}$), of the structure can be located either all to the left or all to the right or some to the left and some to the right of critical region, a_e . The further away from critical region, a_e , the eigen-periods are moved, the better response is obtained, but on the other hand, the higher the gain feedback matrix \mathbf{G} becomes. In Fig. 4 the integrated controlled

$$\begin{aligned} f_{ci} &= \lambda_{ic} T_{li}, \quad k_i \geq 1 \text{ or} \\ f_{ci} &= \lambda_{ic} T_{hi}, \quad k_i \leq 1 \quad i=1,2,\dots,r \end{aligned} \quad (10)$$

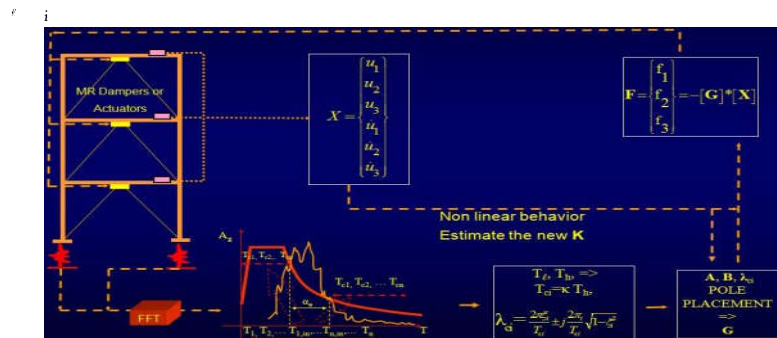


Figure 4 - The integrated controlled system based on the comparison of response spectrum of the incoming earthquake to the design spectrum of structure.

3 Numerical examples

The proposed approach is demonstrated by means of numerical examples, where a singlestory building, is analyzed. The structure is subjected to an Aigio-1995 earthquake record. The mass, stiffness and damping of the uncontrolled building are: 50t, 1.935x10⁵ kN/m and 331 kNs/m, respectively. The period of single degree of freedom system is 0.1 sec. The structure is shown in Fig. 5. F

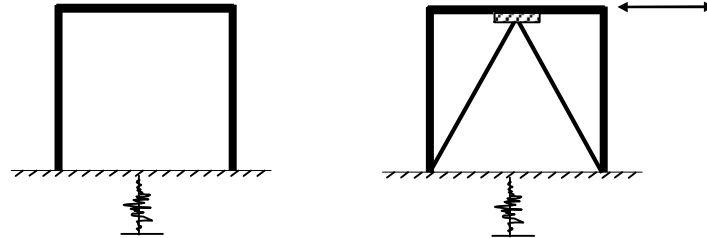


Figure 5 - The uncontrolled and controlled structure subjected to earthquake excitation.

The elastic, the design and the response spectrum of the earthquake as well as the location of the period of the uncontrolled and controlled structure are shown in Fig. 6.

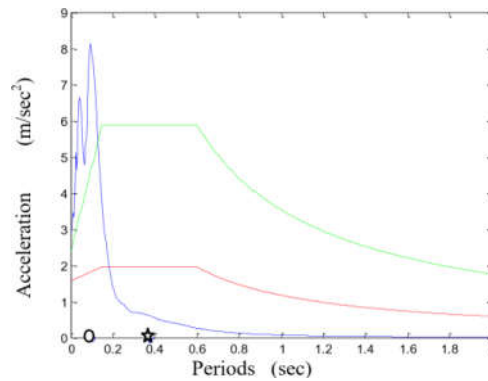


Figure 6 - Elastic, design and the response spectrum of the Aigio 1995 earthquake, the location of the period of the uncontrolled “o” and controlled “*” structure.

Base on the new location of period of structure the feedback matrix was estimated through the pole placement algorithm and the response results, displacement and acceleration of controlled and the uncontrolled structures are presented in Fig. 7.

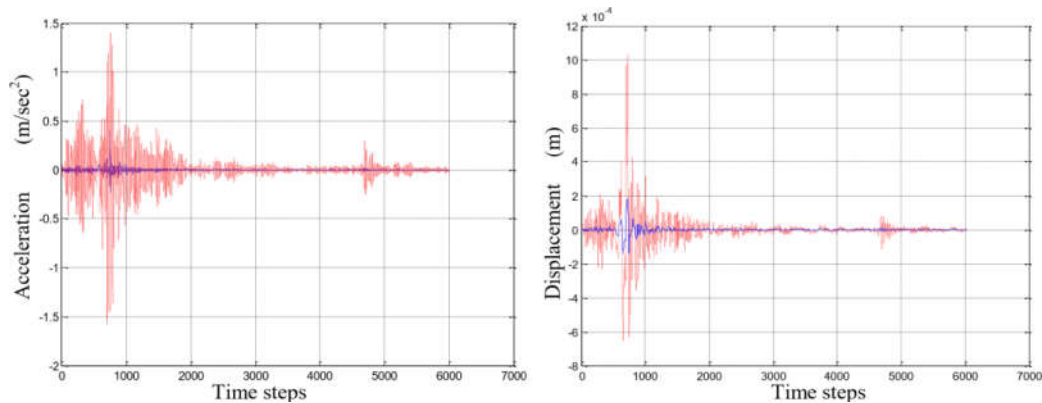


Fig. 7 - Acceleration and displacement of controlled (solid line) and uncontrolled (dash line) structure subjected to Aigio, 1995, earthquake.

The control effort which should be applied to the building to archive the above results is shown in Fig. 8.

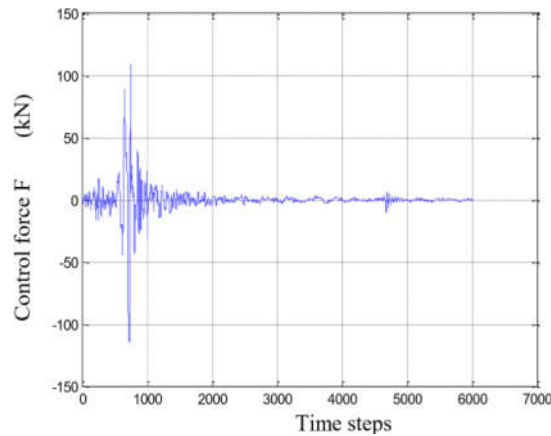


Fig. 8 - The demanded force of the single-story building subjected to Aigio, 1995, earthquake.

The summarized numerical maximum results and the reduction for displacement and acceleration for controlled and uncontrolled structure are presented in table 1. The maximum force needed to achieve the above reductions is also shown in this table.

Table 1: Summarized numerical results

		Controlled	Uncontrolled	Reduction %
u_1	(mm)	0.19	1	80
\ddot{u}_1	(m/sec ²)	0.40	1.58	75
F_1	(kN)	115		

4 Summary and conclusions

A control algorithm which is based on non-resonance theory was investigated. The main idea of this algorithm is to place the eigen-periods of the controlled structure to be outside of the critical region where the response spectrum of earthquake is higher to the design spectrum. From the numerical results it is clear the effectiveness of the control algorithm to the reduction of the response of the structure in terms of both displacement and acceleration. Furthermore the control demand force which should be applied to achieve the reduction in response is kept in acceptable level.

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