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MULTI-PERIOD ASSEMBLY SYSTEMS MODELING WITH UNCERTAINTY LT AND PARTIAL BACKORDERING

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Abstract:-
In this paper deals with material requirement planning for a three levels production and assembly system with several types of components and one type of product, in multi periods. We assume that components lead-times are probabilistic. A MRP approach with periodic order quantity (POQ) policy is used for planning of components. We consider a new approach for modeling MRP when all components lead-times are probabilistic and in this model consider to partial backordering and defect rate for all components in MRP models, which has apparently not been studied before. The objective is minimizing sum of the all components holding cost, final product partial backordering cost, final product holding cost and setup costs. The main policies in this model determine the periodic order quantity, and planned lead-times. Monte-Carlo simulation used to generate numerous scenarios based on the components lead-time, and by using Monte-Carlo simulation, we can find the suitable solution for this problem.

Keywords:- *Planned lead-time, periodic order quantity, uncertainty, Monte-Carlo simulation, Probabilistic lead-time, partial backordering*

1. INTRODUCTION

Material requirement planning (MRP) is a well-known approach to inventory management of dependent demand items. Items that are independently demanded are typically finished goods, while dependently demanded items are typically components and sub-assemblies that are related to an end item by a bill of materials. Thus, MRP is a hierarchical system that provides key information to planners for developing lot-sizing techniques for lower-level components and purchased items, (2) capacity decisions, and (3) sequencing decisions for open orders, while the master production schedule is the main input to an MRP system (see New [11] and Orlicky [12]). In the literature of production planning and inventory control, most papers examine inventory systems where lead time is supposed to be equal to zero or constant. In reality, lead times are rarely constant; unpredictable events can cause random delays. True, in some cases, lead time uncertainties have essentially no effect and therefore can be ignored (or included in lead time demand modeling).

Component requirement planning in assembly systems is crucial for the companies. By optimizing component supplies enterprises can generate large gains in efficiencies. For different reasons (machine breakdowns, transport delays, or quality problems, etc.), the component leadtimes (time of component delivery from an external supplier or processing time for the semifinished product at the previous assembly level) are often random.

The previous states of the art can be find in the following papers. Yeung, Wong, and Ma (1998) propose a review on parameters having an impact on the effectiveness of MRP systems under deterministic or stochastic environment.

Yucesan and De Groote (2000) give a survey on supply planning under uncertainties, but they focus on the impact of the production management under uncertainty on the lead times by observing the service level. Process uncertainties are considered by Koh et al. (2002) and Koh and Saad (2003). Very recently, Mula, Poler, Garcia-Sabater, and Lario (2006b) present a review for production planning under uncertainty. They categorize papers into four modelling approaches (conceptual, analytical, artificial intelligence and imulation).

This is a new survey on supply planning under uncertainties in MRP systems (a first version of this paper has been presented at the 16th IFAC World Congress (Dolgui, Louly, & Prodhon, 2005)). In literature, a number of models exists for dealing with random demand.

Table 1 gives a brief review of main points in some papers regarding supply planning in uncertain environment. The supply planning for the multi-level assembly systems under uncertainty of component lead times has apparently not been studied before. To our knowledge, there is only an approximate study of Axsater [1] for the case of continuous lead-times.

Table 1: Lead-time uncertainly

paper	Criteria	Parameters	Policy	Type of system	Comments
Yano (1987)	Inventory cost	Safety stocks	lot-for-lot		Nonlinear programming
Yano (1987c)	Sum of holding and tardiness costs.	Safety stocks	lot-for-lot	two-level two-component	optimization algorithm
Chu et al. (1993)	Sum of the holding cost for the components and the backlogging cost for the assembled product.	Safety lead-time	lot-for-lot	*One- level* Multi-component	optimal values of the planned times for the single-period problem
Candace Aral Yano b (1987)	Minimizing the sum of inventory holding costs and tardiness costs.	planned lead-times	lot-for-lot	two-level	optimal values of the planned times for the single-period problem
Hegedus and Hopp(2001)	Inventory cost, service level.	Safety lead-time	lot-for-lot	Multi-component	Optimization, minimize inventory costs while ensuring a service level
Ould Louly and Dolgui	Holding and backlogging costs.	Safety lead-	Lot for lot	One- level Multi-	Markovian model for a dynamical

(2004)		time		component multi-period	multi-period planning
Tang and Grubbström (2003)	Stock-out and inventory holding costs.	Safety stocks	lot-for-lot	two-level two-component	Laplace transform
Ould Louly et al. (2008)	Minimize the average holding cost for components while keeping a high customer service level for the finished product.	planned lead-times	lot-for-lot	one-level Multi-component	Branch and Bound
Faicel Hnaien, Xavier Delorme, Alexandre Dolgui (2009)	Minimize the sum of the holding costs for the components of two levels and the backlogging cost for the finished product.	planned lead-times	lot-for-lot	two-level Multi-component	genetic algorithm optimization
Mohamed-Aly Louly AlexandreDolgui (2011)	Sum of the average component holding, finished product backlogging and setup costs.	Planned lead times	POQ	Single level Multi-component	Optimization
H.Sadeghi, A.Makui and M.Heydari (2013)	Minimize the sum of fixed ordering, holding and backlogging costs.	Planned lead times	POQ	multi period serial production system	Optimization

This paper deals with material requirement planning for a three levels production and assembly system with several types of components and one type of product, in multi periods. We assume that components lead-times are probabilistic. A MRP approach with periodic order quantity (POQ) policy is used for the planning of components. a simulation algorithms is used to minimize the sum of the all components holding cost, final product partial backlogging and setup costs.

2. Problem and model description

A multi period supply planning for three-level assembly systems, with multi components is considered in (Fig.1). In this paper, we suppose that, the demand per period is constant. The required quantity of each component is ordered at the beginning of each period, the demands are satisfied at the end of the period. The unit holding cost for each type of component and the unit partial backordering cost for the final product are known. Lead-times for various components' orders are independent and actual lead-time is probabilistic for all components. We used POQ policy for ordering.
In addition we consider partial backordering in the context of batch ordering, which does not consider to it at all.
Also used of breakdown rate for all components for first time and does not consider to it in prior researches.

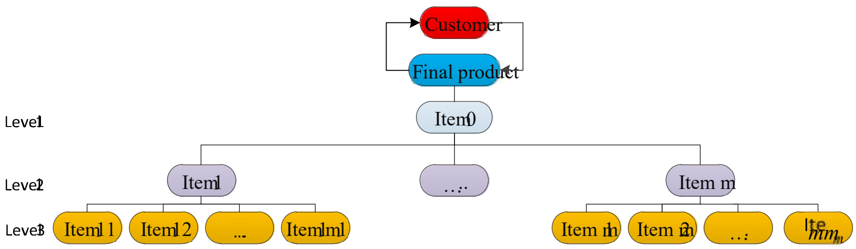

Fig.1: A tree-level assembly system

To take into account the particularities of MRP parameterization, the following assumptions will be considered in this paper:

- Components are ordered from external suppliers to satisfy the customer demand.
- POQ policy is used: components are ordered at every p periods.
- The goal of this model is to search for the optimal values of the parameters p and x.
- Probabilistic lead time for all components
- Demand is constant for all period
- The break down rate is constant for all components

The following notations are used for proposed model

t : Index of period's t=1, 2, 3,…
m: Total number of component in level 2
A : fixed ordering cost,
x_0 : Planned lead-time in level 1,

xi : Planned lead-time for component i in level 2.
xij : Planned lead-time for component j in level 3 for parent i.
ai : Quantity of component i needed to assemble the finished product
D : Demand for final product in period
t_p: components are ordered at every P period in POQ policy
s: the unit selling price
h : Per unit holding cost per time unit for final product
hij : unit holding cost for component j in level 3 for parent i.
cb : the cost to keep a unit backordered for a unit time
α : Defect rate for all components
cg : The goodwill loss on a unit of unfilled demand
c_1 : $(s - c\,p) + cg$: The cost for a lost sale, including the lost profit on that unit and any goodwill loss
β : The fraction of stock-outs that will be backordered
li: Actual lead-time for component i in level 2(random variable with known probability distribution)
lij : Actual lead-time for component j in level 3 for parent i. (random variable with known probability distribution)
$f(lij)$: The probability distribution of lead time for component j in level 3 for parent i.
$C^{*}(x, p)$: The average of total cost in each period

Variables
p: periodicity
x: planned lead-time for final product ($x = (x0, x1, x2...xm, x21,.....x2n)$)

In the model considered, The demand D of finished products per period is constant and the quantities ordered are the same and equal to DP, and a_i units of component i is needed to assemble one finished product. The periodic order quantity (POQ) policy is used, with a periodicity of p periods.
The unit holding cost h of final period, unit backlogging cost b of a finished product per period and setup cost c are known. The distribution of the component i lead-time l_i is also known.
In this paper, an approach is proposed to optimize the planned lead-time x and the periodicity p of POQ policy minimizing the sum of setup and holding costs while respecting a service level constraint. The method suggested takes into account the fact that the actual lead times are random.

3. Mathematical model

The lead-time is assumed probabilistic. The planned lead-time for component i in level 3 is $x_{ij,}$ planned lead-time for component i in level 2 is x_i i 1,2,3...m and planned lead-time for level 1 is x_0 .The order for product is made at the beginning of the periods 1, p+1, 2p+1… and there is no order made in other periods. Order quantities are constant and equal to PD. (See Fig. 2)

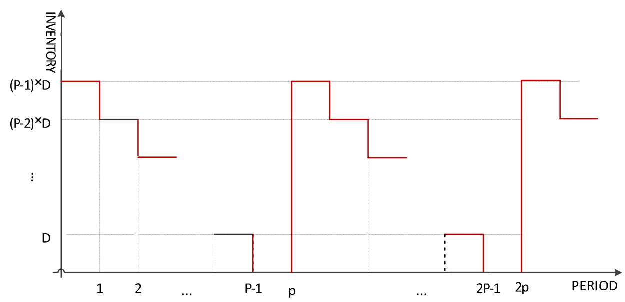

Fig .2: An illustration of the planning problem for final product

Taking in to account the fact that the different components on the same level do not arrive at the same time, there are stocks at levels 1 and 2.If the final product is assembled after the due date, there is backlog and therefore we have holding and backordering cost and if product is assembled before the due date, there is stocks and we have holding cost (see Fig. 3).

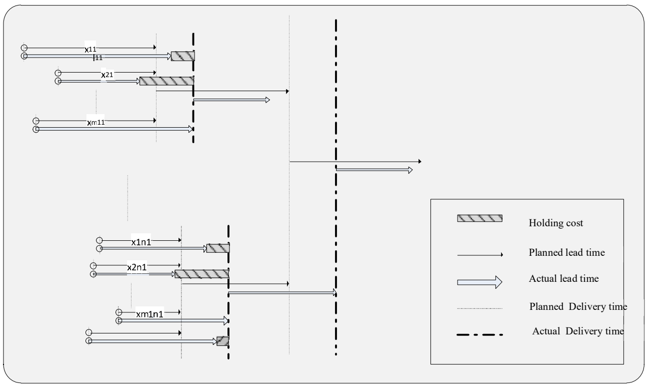

Fig. 3. An illustration of the planning problem.

In this paper, it is assumed that during manufacture or assembly, the percentage of components is discarded as waste. Therefore, the cost of production / assembly and purchase costs can be increases. Because at each stage of production / assembly percentage of components are wasted. The objective is to find planning lead-time for all components and priority order, in order to minimize the total of the holding costs for the components and final product and backlogging cost for the final product.

The orders for products are made at the beginning of the periods 1, p+1, 2p+1,… and ordered for the needs of P period which is equal to PD for each ordering.

According to Fig.3, we have holding cost for some component in level 2, and for final product have holding, and backordering cost. Therefore, the costs of this model include holding cost for all components and final product, backordering cost for final product and fix order cost for each ordering.
Because of probabilistic lead-time, there are three states in action:

1. The planned lead-time for first level equal to actual lead-time in this level (see Fig .2) This state has not backorder and model costs are equal to:

$$C_1(x,p)=\left[\overbrace{A}^{\text{Setup cost}}+\overbrace{(p-1)hD+(p-2)hD+\ldots+2hD+hD}^{\text{Final product Holding cost}}+D\overbrace{\frac{\sum_{i=1}^{m}h_i a_i(x_i+k-k_i-l_i)}{\alpha\times\alpha\times\alpha}}^{\text{Items Holding cost in level 2}}+\overbrace{D\sum_{i=1}^{m}\sum_{j=1}^{m_i}h_{ij}\frac{a_{ij}(x_{ij}+k_i-l_{ij})}{\alpha\times\alpha}}^{\text{Items Holding cost in level 3}}\right]\times p(l_0=x_0)$$

$$\Rightarrow\ C_1(x,p)=\left[A+\frac{p(p-1)}{2}hD+D\sum_{i=1}^{m}h_i\frac{a_i(x_i+k-k_i-l_i)}{\alpha\times\alpha\times\alpha}+D\sum_{i=1}^{m}\sum_{j=1}^{m_i}h_{ij}\frac{a_{ij}(x_{ij}+k_i-l_{ij})}{\alpha\times\alpha}\right]\times p(l_0=x_0) \qquad (2)$$

Where $k=Max(l_i-x_i)$, $k_i=\underset{j}{Max}(l_{ij}-x_{ij})$

2. The planned lead-time for first level is smaller than to actual lead-time for first level. If the final product is assembled after the due date, there exists backlog (see Fig .4).

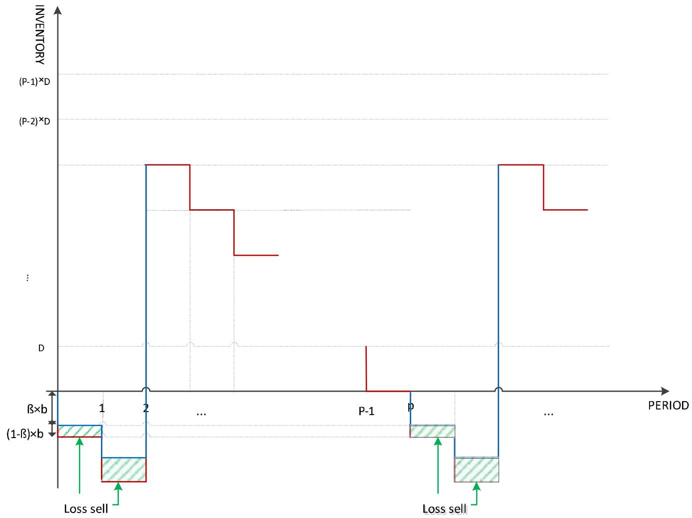

Fig. 4. The planned lead-time is smaller than to actual lead-time for first level

In this state, the cost is equal to:

$$
\begin{aligned}
C_2(x,p) = & \left[\overbrace{A}^{\text{Setup cost}} + \overbrace{\frac{(p-(l_0-x_0+k))(p-1-(l_0-x_0+k))}{2\times\alpha}hD}^{\text{Final product Holding cost}} \right] \times P(l_0 > x_0 - k) \\
& + \beta \left[\overbrace{c_b \times D \frac{(l_0-x_0+k)(l_0-x_0+k+1)}{2\times\alpha}}^{\text{Final product Backordering cost}} \right] \times P(l_0 > x_0 - k) \\
& + (1-\beta) \left[\overbrace{c_1 \times D \frac{(l_0-x_0+k)(l_0-x_0+k+1)}{2\times\alpha}}^{\text{Final product Backordering cost}} \right] \times P(l_0 > x_0 - k) \\
& + \overbrace{D\sum_{i=1}^{m} h_i a_i \frac{(x_i+k-k_i-l_i)}{\alpha\times\alpha}}^{\text{Items Holding cost in level 2}} + \overbrace{D\sum_{i=1}^{m}\sum_{j=1}^{m_i} h_{ij} \frac{a_{ij}(x_{ij}+k_i-l_{ij})}{\alpha\times\alpha\times\alpha}}^{\text{Items Holding cost in level 3}} P(l_0 > x_0 - k) \qquad (3)
\end{aligned}
$$

3. The planned lead-time for first level is bigger than the actual lead-time for first level (see Fig .5).

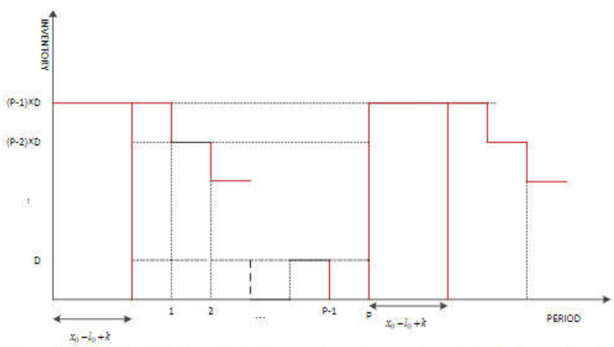

Fig. 5. The planned lead-time is bigger than the actual lead-time for first level

In this state, the cost is equal to:

$$C_3(x,p)=\left[\overbrace{A}^{\text{Setup cost}}+\overbrace{\frac{p(p-1)}{2}hD+\frac{hpD(l_0-x_0+k)}{\alpha}}^{\text{Final product Holding cost}}\right]\times P(l_0<x_0-k) \quad (4)$$

$$+\left[\overbrace{D\sum_{i=1}^{m}h_i\frac{a_i(x_i+k-k_i-l_i)}{\alpha\times\alpha}}^{\text{Items Holding cost in level 2}}+\overbrace{D\sum_{i=1}^{m}\sum_{j=1}^{m_i}h_{ij}\frac{a_{ij}(x_{ij}+k_i-l_{ij})}{\alpha\times\alpha\times\alpha}}^{\text{Items Holding cost in level 3}}\right]\times P(l_0<x_0-k)$$

Where $k=Max(l_i-x_i)$, $k_i=\underset{j}{Max}(l_{ij}-x_{ij})$

Total costs are expressed as follows:

$$\begin{aligned}C(x,p)&=C_1(x,p)+C_2(x,p)+C_3(x,p)=\\&A+\left(\frac{p(p-1)}{2}hD+\frac{hpD(x_0-l_0-k)}{\alpha}\right)\times P(l_0\le x_0-k)+\beta\left[c_b\times D\frac{(l_0-x_0+k)(l_0-x_0+k+1)}{2\times\alpha}\right]\times P(l_0>x_0-k)\\&+(1-\beta)\left[c_1\times D\frac{(l_0-x_0+k)(l_0-x_0+k+1)}{2\times\alpha}\right]\times P(l_0>x_0-k)+\frac{hD(p-l_0+x_0-k)(p-l_0+x_0-k-1)}{2\times\alpha}\times P(l_0>x_0-k)\\&+p\left[D\sum_{i=1}^{m}h_i\frac{a_i(x_i+k-k_i-l_i)}{\alpha\times\alpha}+D\sum_{i=1}^{m}\sum_{j=1}^{m_i}h_{ij}\frac{a_{ij}(x_{ij}+k_i-l_{ij})}{\alpha\times\alpha\times\alpha}\right] \qquad (5)\end{aligned}$$

As shown in the previous proposition, the cost of a single period kp + r is a random variable. To study the considered multi- period problem, explicit closed forms should be obtained for the average cost and the average number of shortages on the infinite horizon, i.e. for the following expressions:

$$\hat{C}(x,p)=\lim_{w\to\infty}\sum_{i=1}^{w}\frac{C(x,p)}{p\times r}\Rightarrow$$

Then by using Eq. (6), the expressed unit cost will be as follows

$$\hat{C}(x,p)=\frac{A}{p}+\left[\begin{aligned}&\frac{(p-1)}{2}hD+hD\times\frac{E[(x_0-l_0-k)]}{\alpha\times\alpha}+p\left[D\sum_{i=1}^{m}h_ia_i\frac{(x_i+k-k_i-l_i)}{\alpha\times\alpha}+D\sum_{i=1}^{m}\sum_{j=1}^{m_i}h_{ij}\frac{a_{ij}(x_{ij}+k_i-l_{ij})}{\alpha\times\alpha\times\alpha}\right]\\&+\frac{D}{2p}\left(h\frac{[(l_0-x_0+k)^2+(l_0-x_0+k)]}{\alpha}+\beta\left[c_b\frac{(l_0-x_0+k)(l_0-x_0+k+1)}{\alpha}\right]+(1-\beta)\left[c_1\frac{(l_0-x_0+k)(l_0-x_0+k+1)}{\alpha}\right]\times P(l_0>x_0-k)\right)\end{aligned}\right] \quad (6)$$

The cost $C\hat{}(x,p)$ is a random variable (because $l0,l1...lm$ and k are random variables

Noted that x is planned lead-time for $p\times D$ components and l is actual lead-time for D

Componentsin one period then in the all equation x equal to $\frac{x_p}{p}$ which $_{p}x$ is planned lead-time x_p periods.

4. Simulation method

The study objective is to integrate reliability analysis with expansion planning and dispatching decisions. To achieve this objective, we propose a simulation based optimization approach.

Monte Carlo methods provide a good means for generating starting points for optimization problems that are non-convex. In its simplest form, a Monte Carlo method generates a random sample of points in the domain of the function. We use our favorite minimization algorithm starting from each of these points, and among the minimizers found, we report the best one. By increasing the number of Monte Carlo points, we increase the probability that we will find the global minimizer. Thereby, the Monte Carlo simulation, as a random numerical simulation, becomes a validated method of treating a complex problem, which cannot be solved by general equations, or experimental analysis methods. The principle of the Monte Carlo simulation for statistical tolerance analysis is to use a random generator to simulate the variations of dimension tolerances.

We generate numerous scenarios considering the lead-time of components. Each scenario represents a random answer of components production time. For each scenario, components lead-time is generated randomly by lead-time distributions.

The pseudo code of this algorithm is as follows:

Step 1:input all parameters include A,h,b, h_{ij} and set p=1

Step2: set j=1, $c_1 = \infty$

Step 3: generate random data for lead-time

$$x_0 \leftarrow Random\ data\ with\ l_0\ destribution$$
$$x_i \leftarrow Random\ data\ with\ l_i\ destribution$$
$$x_{ij} \leftarrow Random\ data\ with\ l_{ij}\ destribution$$

Step 4: Calculate total cost for this generation

Step5: If $C(p,x) < C_p$ then $C_p \leftarrow C(p,x)$ and $x_p \leftarrow x$

$j \leftarrow j+1$, save C_p and x_p

Step 6: If the stopping criterion for j is met, stop and return C_p and x_p

Step 7: If $C_p < C_{p-1}$ then $p \leftarrow p+1$

then go to step 2

Else

C_{p-1} is minimum cost and x_{p-1} is the best answer

End

Example:

Consider the assembly system with two levels, which there are 3 components in level 2 and 9 components in level 3. The probability distributions for all components and unit holding cost are shown in table 1. And other parameters are: A=100 , b=5, h=10.

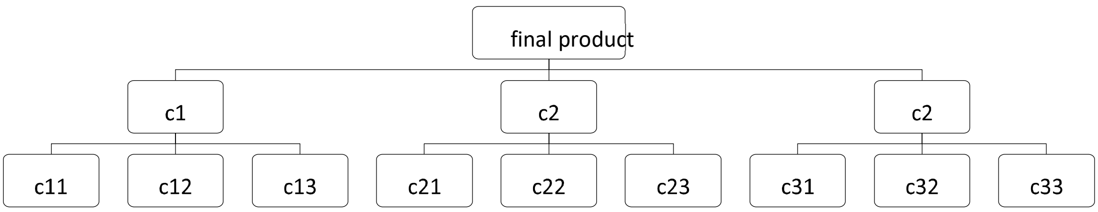

Fig6: components level in example 1

Table2: the probability distribution and unit holding cost of lead-time for all components

level	components	Lead time distribution	Unit holding cost
1	Final product	$U(5,15)$	20
2	1	$U(1,5)$	10
2	2	$U(2,7)$	12
2	3	$U(3,8)$	15
3	11	$U(5,10)$	4
3	12	$U(4,8)$	10
3	13	$U(5,12)$	1
3	21	$U(3,5)$	2
3	22	$U(3,10)$	12
3	23	$U(2,9)$	15
3	31	$U(9,12)$	9
3	32	$U(7,12)$	2
3	33	$U(5,10)$	5

Solution:

We generate 10,000 scenarios considering the lead-time of components. Each scenario represents a random answer of components production time. For each scenario, components lead-time is generated randomly by lead-time distributions. In each scenario, the total cost calculated and in result find minimum cost in 10,000 scenarios. The result of this simulation represent as follows:

Table3: simulation solution for example 1

P	1	2	3	**4**	5	6	7	8	9
Minimum total cost	3269.9	2947.5	2566.7	**2595.7**	2867.4	2844.33	2928.56	2978.66	3350.3
Total holding cost	2917.1	2639.9	2275.2	**2310**	2512.5	2413.78	2620.776	2638.18	2303.3
Total backordering cost	15.75	7.878	31.51	**23.63**	66.18	144.4	13.5	41.3636	710.8
x_0	14	28	36	**48**	65	60	91	96	81
x_1	3	4	12	**16**	15	30	21	32	18
x_2	6	8	24	**32**	15	54	35	24	63
x_3	3	12	15	**20**	15	24	21	48	27
x_{11}	7	14	18	**24**	35	48	42	48	45
x_{12}	5	14	12	**16**	40	42	42	64	54
x_{13}	8	10	21	**28**	25	48	49	48	72
x_{21}	5	10	9	**12**	20	30	28	40	36
x_{22}	5	6	9	**12**	40	36	63	64	27
x_{23}	9	4	9	**12**	45	18	42	64	18
x_{31}	10	22	36	**48**	55	54	70	88	99
x_{32}	10	16	36	**48**	35	66	56	96	81
x_{33}	10	16	21	**28**	35	54	63	48	81

The best answer in this simulation obtained when periodicity order equal to four (see Table 2). Therefore, planning cycle equal to four period and we should produce final product for four period.

5. Conclusion

In this paper considered with a model for optimizing the planned lead-time and order periodicity for production in multi-level production system with random lead-time for all components. At first, we model this multi-level multi period MRP with considering to uncertainly lead-time of components. We assume that when the components are made, it is possible that same of them are discarded during manufacture or assembly. Moreover, assume the backorder is partial.

Then generate numerous scenarios considering the lead-time of components. Each scenario represents a random answer of components production time. For each scenario, components lead-time is generated randomly by lead-time distributions. For each interaction, the total cost should be calculate and compared with prior total cost, if it is smaller than saved this cost. The proposed a simulation model to minimize the sum of the average holding cost, backlogging product and setup costs. This method, also can calculate the cost of the Lot for Lot policy. The cost of Lot for Lot order policy is when P equal to one. In this paper, a problem is solved to show the efficacy of cost parameter's on optimal planned lead-time and periodicity time. As a future research, one can consider the multi-level uncertainly MRP model, which cannot consider to it at all.

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